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Basic concepts in metrology

Terminology relating to the uncertainty of measurement and the assessment of measuring instruments and measuring equipment

DIN 1319 Part 3

Grundbegriffe der Messtechnik; Begriffe für die Messunsicherheit und für die Beurteilung von Messgeräten und Messeinrichtungen

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As it is current practice in standards published by the International Organization for Standardization (ISO), the comma has been used throughout as a decimal marker.

Contents

1 Field of application and scope

This standard applies both to the evaluation and assessment of measurements of physical quantities (see DIN 1319 Part 1 and DIN 1313) and to the assessment of the measuring instruments and measuring equipment used for this purpose.

It is the object of this standard to lay down the terminology for the uncertainty of measurement and also rules for its numerical determination for the case of a single (more especially directly) measured physical quantity; to cover other cases, e.g. a physical quantity as a function of two or more other quantities, a further standard is in preparation. Moreover in clause 8 of this standard basic concepts for assessing measuring instruments and measuring equipment are defined.

2 General principles

It is the object of every measurement to determine the true value of a measurand. Because of the influences mentioned in subclause 3.1 and affecting the measurement, errors of measurement occur (hereafter usually referred to in brief as "errors"). They are the reason why it is impossible to find the true value x_w. It is therefore assumed notionally that the values obtained from several individual measurements of a series of measurements, namely the measured values x_i, are realizations of a random variable X. This random variable X obeys a probability distribution characterized in particular by two parameters which are the expectation μ and the standard deviation σ. In the absence of systematic errors (see subclause 3.3) the expectation μ agrees with the true value x_w of the measurand. The standard deviation σ is a measure of variability for the random error of an individual measured value (see subclause 3.2) from the expectation of the measurand.

The parameters μ and σ of the probability distribution are generally not known. The problem is to determine estimates for them from a series of measurements. Usually the arithmetic mean $\bar{x}$ (see subclause 5.1) is used as the estimate for μ and the (empirical) standard deviation s of the series of measurements (see subclause 5.2) as the estimate for σ. Because the measured values are realizations of a random variable, $\bar{x}$ will deviate from μ and s from σ in a random manner.

On the basis of an assumption concerning the type of distribution of the measured values (normal distribution is assumed in this standard) it is possible with the aid of $\bar{x}$ and s to state a confidence interval which, with a given probability – the confidence level $(1 - \alpha)$ – covers the expectation μ (see subclause 5.3). This confidence interval allows for the influence of the random errors on the result of the measurement (see DIN 1319 Part 1).

Known systematic errors are eliminated by making corrections (see subclause 3.3.2). An attempt is made to deal with unknown systematic errors (see subclause 3.3.3) by widening the confidence interval. How the confidence interval is widened depends on assumptions derived empirically regarding the unknown systematic errors (see subclause 6.2).

The final result of measurement from a series of measurements consists of the mean value, which has been corrected for known systematic errors, combined with an interval in which the true value x_w of the measurand is presumed to lie. The difference between the upper limit of this interval and the corrected mean value or the difference between the corrected mean value and the lower limit of the interval is termed the uncertainty of measurement. Usually, but not always, the two differences are equal in value (cf. subclauses 6.3 and 7.1).

Continued on pages 2 to 15

3 Causes and types of errors of measurement

3.1 Causes of errors of measurement

Every measured value and hence every result of measurement of a measurand is *influenced* by imperfections of the measuring instruments and measuring equipment (including material measures), of the method of measurement, and of the measurement object, also by the environment and the observers; moreover these influences are also subject to variation with time.

Environmental influences to be taken into account are local differences and variations with time of, for example, temperature, atmospheric pressure, humidity, voltage, frequency, external electrical or magnetic fields (see also subclause 8.3.1, Note 2 regarding influence quantities).

Influences due to the observer depend on the qualities and capabilities of the observers (e. g. vigilance, practice, visual acuity, estimating ability).

Apart from such influences, a result of measurement may be *falsified* through mistakes on the part of the observers, through choice of a method of measurement or evaluation which is unsuitable for determining the measurand observed, and also by non-observance of known disturbing influences. Faulty practices of such kinds are not dealt with in this standard.

3.2 Random errors, variablity of measured values

Non-controllable, non-unilaterally acting influences during a number of measurements on the same measurement object within a series of measurements lead to a *variability* of the measured values about the mean value of the series of measurements (see subclause 5.1) and hence to random errors of the measured values from the true value.

The random variability may be characterized by suitable statistical quantities and stated numerically through their estimates (see clause 5). The prerequisite for reliable estimates is that a sufficient number of measured values obtained under repeatability conditions (see subclause 4.1) is available. The random variability of the measured values – in conjunction with the unknown systematic errors (see subclause 3.3.3) – makes a result of measurement *uncertain*.

The value taken as the estimate for the true value is the mean, so long as no systematic errors arise.

Note. When the same observer repeats on the same measurement object a measurement of the same measurand using the same measuring instrument under the same conditions, the individual measured values will deviate from one another, they will exhibit "random variability" (see also clause 5).

The variability of the individual values in a series of measurements may also be brought about by the fact that the measurement object itself changes while the measurement is proceeding, i.e. its property undergoing measurement, namely the measurand, is subject to random fluctuations. In this case also, which commonly arises in practice, the forming of the mean and of the standard deviation is appropriate, and in such cases also a confidence interval can be stated which now takes account of the variability of the measurand.

A major source of variability may also be the inhomogeneity of the measurement object; the result of measurement may often only be obtainable as a mean of a larger number of different individual measurements on the same measurement object, e. g. the hardness of a steel component.

3.3 Systematic errors

3.3.1 General

There are

a) systematic errors which throughout the measurement maintain a constant magnitude and a given sign (either plus or minus) (e. g. as a result of incorrect adjustment of the measuring instrument);

b) systematic errors which vary with time and are brought about by causes which effect a change in the measurand in a certain direction (e. g. directional thermal response, wear, ageing). As far as possible such changes with time are to be avoided during the measurement.

Systematic errors are present in every result of measurement and cannot be discovered under repeatability conditions (see subclause 4.1).

Note. Strict differentiation between random errors and unknown systematic errors (see subclause 3.3.3) is not always practicable and meaningful. Under certain preconditions, e. g. in round robin tests with a sufficient number of participants, systematic errors also can be dealt with like random errors.

3.3.2 Known systematic errors

Known systematic errors – both constant and varying with time – shall be allowed for by making corrections as specified in subclause 8.2.5. In this way a *corrected measured value* is obtained. If a measured value affected by systematic errors is left uncorrected, the result will be *incorrect*.

Note. The known systematic errors include, for example, systematic errors of measuring instruments determined by calibration; these are allowed for by correcting as indicated in subclause 8.2.5.

3.3.3 Unknown systematic errors

There are also systematic errors which are inferred or made apparent on the basis of experimental experience, but whose magnitude and sign cannot be definitively stated, or are entirely unknown. Such unknown systematic errors can, however, in many cases be *estimated* (by non-statistical methods); in such cases they have to be additionally taken into account in a suitable manner when calculating the uncertainty of measurement (see subclauses 6.2 and 6.3). In addition, there are also unknown systematic errors which are not capable of estimation.

Note. Unknown systematic errors may be caused by the fact that a measuring instrument has an unknown error or by failure to make allowance for unavoidable disturbing influences affecting a method of measurement.

Example:

Heat losses through conduction when making caloric and temperature measurements. In such cases elucidation can only be obtained by applying measuring instruments or methods of measurement of a different or superior kind.

4 Test conditions

Each time before an assessment of the measured values of a measurand is made, a check shall be carried out to determine whether the measurements have been performed under the same test conditions and independently of one another. It has proved desirable to single out the following *two boundary cases* from the many kinds of possible test conditions (see DIN 55 350 Part 13 and DIN ISO 5725).

4.1 Repeatability conditions

Repeatability conditions exist when the same observer carries out measurements by a specified method of measurement on the same measurement object under the same test conditions (same measuring instrument, same laboratory) a number of times within short periods of time.

The standard deviation under repeatability conditions is termed the *repeatability standard deviation* σ_r. From a known repeatability standard deviation σ_r it is possible to calculate the value below which the magnitude of the difference of two measured values under repeatability conditions can be expected to lie with a probability of 95 %; this is the *repeatability*.

$$r = 1{,}96\sqrt{2}\,\sigma_r \approx 2{,}77\,\sigma_r$$

(DIN ISO 5725 uses the numerical value 2,83 ($\approx 2 \cdot \sqrt{2}$) instead of 2,77).

Under repeatability conditions the same systematic errors usually occur with each measurement. They are therefore not determinable from the series of repeatability measurements.

4.2 Reproducibility conditions

Reproducibility conditions exist when different observers carry out measurements by a specified method of measurement on the same measurement object under different test conditions (different measuring instruments, different measurement locations or laboratories) at different times.

The standard deviation under reproducibility conditions is termed the *reproducibility standard deviation* σ_R. From a known reproducibility standard deviation σ_R it is possible to calculate the value below which the magnitude of the difference of two measured values under reproducibility conditions can be expected to lie with a probability of 95 %; this is the *reproducibility*

$$R = 1{,}96\sqrt{2}\,\sigma_R \approx 2{,}77\,\sigma_R$$

(DIN ISO 5725 uses the numerical value 2,83 ($\approx 2 \cdot \sqrt{2}$) instead of 2,77).

Under reproducibility conditions, comparison of the measured values from the different laboratories reveals systematic errors differing from one another which are not directly determinable in any single laboratory.

Example:

With many standardized methods of measurement in the field of mineral oil testing the reproducibility standard deviation σ_R is about twice as large as the repeatability standard deviation σ_r.

Note 1. As a qualitative generic term for the (quantitative) magnitudes repeatability standard deviation, repeatability, reproducibility standard deviation, and reproducibility the term "precision" is often used, see DIN 55 350 Part 13.

Note 2. Since the values of the repeatability standard deviation and reproducibility standard deviation are determined in practice in round robin tests of comparatively long duration (also involving a larger number of participating laboratories), the empirical standard deviations s_r and s_R (see subclause 5.2) can be regarded as adequate estimates for σ_r and σ_R for the purpose of statistical evaluation.

5 Theoretical determination of the random variability of measured values of a series of measurements

5.1 Arithmetic mean $\bar{x}$

If, in a series of measurements, n individual measured values $x_1, \ldots, x_i, \ldots, x_n$, which are independent of one another, are measured under repeatability conditions (see subclause 4.1) the *arithmetic mean* of these n individual values, referred to in brief as the mean $\bar{x}$ (pronounced as x bar) is given by

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad (1)$$

$\bar{x}$ being an estimate for the expectation μ.

Note. Individual values are *independent* of one another if successive measurements, and hence the individual measured values obtained, are not influenced by those which have gone before.

5.2 (Empirical) standard deviation s, coefficient of variation v

The most important operand for the numerical determination of the random variability of n individual values of a series of measurements about their mean $\bar{x}$ is the *(empirical) standard deviation s*:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2} = \sqrt{\frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2\right]} \qquad (2)$$

The square of the standard deviation is termed the *variance* s^2 or σ^2. The (empirical) standard deviation s is the positive root of the variance s^2 and an estimate for the standard deviation σ.

Instead of the (empirical) standard deviation s, the (empirical) coefficient of variation v is also used. v is often expressed in %. The following applies for $\bar{x} \neq 0$:

$$v = \frac{s}{|\bar{x}|} \qquad (3)$$

Note. The plotting of the cumulative sum curve of individual values obtained also leads to an assessment of the results of tests. In this connection reference is made to DIN 55 302 Part 1 and Part 2 "Statistical evaluation methods; mean and variability".

5.3 Confidence limits and confidence interval for expectation μ

5.3.1 General

It shall not be assumed (see clause 2) that the mean $\bar{x}$ is equal to the expectation μ or the true value x_w, not even if no systematic errors are present. It is possible, however, to state an interval about the mean $\bar{x}$ which has been corrected to eliminate known systematic errors (see subclause 3.3.2). This interval embraces the expectation with a given probability $(1-\alpha)$. The limits of this interval are termed the *confidence limits* for the expectation and the interval itself is termed the *confidence interval* for the expectation, assigned to the *confidence level* $(1-\alpha)$. The chosen confidence level $(1-\alpha)$ must therefore always be stated in connection with the confidence interval.

If nothing to the contrary is agreed, the confidence level $1-\alpha = 95$ % shall be used (see ISO 3534).

Note. The confidence level $(1-\alpha)$ was previously often called the "confidence probability" and denoted by the symbol P.

With regard to the use of preferred values of the confidence level in practical applications, the following should be noted: In physics and in surveying practice, calculations are largely made with the simple magnitude of the standard deviation, and the low confidence level of $1-\alpha = 68{,}26$ % which this involves is deliberately accepted. The uncertainty of the natural physical constants is likewise indicated by the simple standard deviation. In biology for a long time past the high confidence level $1-\alpha = 99{,}73$ % (3σ) has been deemed appropriate; more recently, round figures, e.g. $1-\alpha = 99$ %, are preferred internationally. In industry wide use is made, also internationally, of the confidence level $1-\alpha = 95$ %. This is the basis, for example, of all ASTM Standards [4] and of the German mineral oil standards.

The question as to which confidence level to work with depends on the problem concerned or on agreement; a general ruling covering all sectors is pointless. If nothing is stated about the confidence level it should always be permissible to assume a confidence level of $1-\alpha = 95$ %.

In this standard it is taken that the measured values derive from a normal distribution and are obtained independently of one another.

When calculating the confidence limits a distinction shall be made between cases in which the standard deviation σ (see clause 2) is unknown (e.g. when novel kinds of test are involved) and cases in which it is adequately known from experience gained with earlier measurements.

[4] ASTM = American Society for Testing and Materials

Table 1. **Values of t and $t/\sqrt{n}$ for different values of the confidence level $(1-\alpha)$**

Number n of individual values	$1-\alpha=68{,}26$ %		$1-\alpha=90$ %		$1-\alpha=95$ %		$1-\alpha=99$ %		$1-\alpha=99{,}5$ %		$1-\alpha=99{,}73$ %	
	t	$t/\sqrt{n}$	t	$t/\sqrt{n}$	t	$t/\sqrt{n}$	t	$t/\sqrt{n}$	t	$t/\sqrt{n}$	t	$t/\sqrt{n}$
2	1,84	1,30	6,31	4,46	12,71	8,98	63,66	45,01	127,32	90,03	235,8	166,7
3	1,32	0,76	2,92	1,69	4,30	2,48	9,93	5,73	14,09	8,13	19,21	11,09
4	1,20	0,60	2,35	1,18	3,18	1,59	5,84	2,92	7,45	3,73	9,22	4,61
5	1,15	0,51	2,13	0,95	2,78	1,24	4,60	2,06	5,60	2,50	6,62	2,96
6	1,11	0,45	2,02	0,82	2,57	1,05	4,03	1,65	4,77	1,95	5,51	2,25
8	1,08	0,38	1,90	0,67	2,37	0,84	3,50	1,24	4,03	1,42	4,53	1,60
10	1,06	0,34	1,83	0,58	2,26	0,71	3,25	1,03	3,69	1,17	4,09	1,29
13	1,05	0,29	1,78	0,49	2,18	0,60	3,05	0,85	3,43	0,95	3,76	1,04
20	1,03	0,23	1,73	0,39	2,09	0,48	2,86	0,64	3,17	0,71	3,45	0,77
30	1,02	0,19	1,70	0,31	2,05	0,37	2,76	0,50	3,04	0,56	3,28	0,60
32	1,02	0,18	1,70	0,30	2,04	0,36	2,74	0,49	3,02	0,53	3,26	0,58
50	1,01	0,14	1,68	0,24	2,01	0,28	2,68	0,38	2,94	0,42	3,16	0,45
80	1,00	0,11	1,66	0,19	1,99	0,22	2,64	0,30	2,89	0;32	3,10	0,35
100	1,00	0,10	1,66	0,17	1,98	0,20	2,63	0,26	2,87	0,29	3,08	0,31
125	1,00	0,09	1,66	0,15	1,98	0,18	2,62	0,23	2,86	0,26	3,07	0,27
200	1,00	0,07	1,65	0,12	1,97	0,14	2,60	0,18	2,84	0,20	3,04	0,21
Over 200 (then $t = t_\infty$)	1,00	$\frac{1{,}00}{\sqrt{n}}$	1,65	$\frac{1{,}65}{\sqrt{n}}$	1,96	$\frac{1{,}96}{\sqrt{n}}$	2,58	$\frac{2{,}58}{\sqrt{n}}$	2,81	$\frac{2{,}81}{\sqrt{n}}$	3,00	$\frac{3{,}00}{\sqrt{n}}$

Table 2. Confidence limits and confidence interval for the expectation μ with known standard deviation σ

Confidence level $(1-\alpha)$ in %	Lower confidence limit	Upper confidence limit	Confidence interval
68,26	$\bar{x}-\frac{\sigma}{\sqrt{n}}$	$\bar{x}+\frac{\sigma}{\sqrt{n}}$	$\bar{x}-\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x}+\frac{\sigma}{\sqrt{n}}$
90,0	$\bar{x}-\frac{1{,}65\,\sigma}{\sqrt{n}}$	$\bar{x}+\frac{1{,}65\,\sigma}{\sqrt{n}}$	$\bar{x}-\frac{1{,}65\,\sigma}{\sqrt{n}} \leq \mu \leq \bar{x}+\frac{1{,}65\,\sigma}{\sqrt{n}}$
95,0	$\bar{x}-\frac{1{,}96\,\sigma}{\sqrt{n}}$	$\bar{x}+\frac{1{,}96\,\sigma}{\sqrt{n}}$	$\bar{x}-\frac{1{,}96\,\sigma}{\sqrt{n}} \leq \mu \leq \bar{x}+\frac{1{,}96\,\sigma}{\sqrt{n}}$
99,0	$\bar{x}-\frac{2{,}58\,\sigma}{\sqrt{n}}$	$\bar{x}+\frac{2{,}58\,\sigma}{\sqrt{n}}$	$\bar{x}-\frac{2{,}58\,\sigma}{\sqrt{n}} \leq \mu \leq \bar{x}+\frac{2{,}58\,\sigma}{\sqrt{n}}$
99,5	$\bar{x}-\frac{2{,}81\,\sigma}{\sqrt{n}}$	$\bar{x}+\frac{2{,}81\,\sigma}{\sqrt{n}}$	$\bar{x}-\frac{2{,}81\,\sigma}{\sqrt{n}} \leq \mu \leq \bar{x}+\frac{2{,}81\,\sigma}{\sqrt{n}}$
99,73	$\bar{x}-\frac{3\,\sigma}{\sqrt{n}}$	$\bar{x}+\frac{3\,\sigma}{\sqrt{n}}$	$\bar{x}-\frac{3\,\sigma}{\sqrt{n}} \leq \mu \leq \bar{x}+\frac{3\,\sigma}{\sqrt{n}}$

5.3.2 Confidence limits and confidence interval with unknown standard deviation σ

In many practical cases only the (empirical) standard deviation s of a series of measurements with n individual measured values is known (equation 2). In such cases the confidence limits for the expectation μ, which are disposed symmetrically relative to the mean, are given by

$$\left.\begin{aligned} &\text{Upper confidence limit:} \quad \bar{x}+\frac{t}{\sqrt{n}}\,s \\ &\text{Lower confidence limit:} \quad \bar{x}-\frac{t}{\sqrt{n}}\,s \end{aligned}\right\} \quad (4)$$

The factor t depends on the chosen confidence level $(1-\alpha)$ and also on the number n of the individual values. For the six values $1-\alpha$ = 68,26 %; 90,0 %; 95,0 %, 99,0 %; 99,5 % and 99,73 % the assigned values of the factor t (Student t-distribution) and $t/\sqrt{n}$ are listed in table 1. Table 1 shows that when σ is unknown and the number n is small, a wide confidence interval has to be accepted.

Note. Instead of the number n of individual values in column 1 of table 1, in other tables the values for t are often stated as a function of the degree of freedom $f = n - 1$.

5.3.3 Confidence limits and confidence interval with known standard deviation σ

When the standard deviation σ is adequately known for practical purposes through experience gained with earlier measurements, the expressions for the confidence limits and the confidence interval for the expectation μ in the case of n individual measurements are shown in table 2.

6 Uncertainty of measurement u

The result of measurement from a series of measurements is the mean $\bar{x}$, corrected for the known systematic errors, combined with an interval in which the true value of the measurand is presumed to lie. The difference between the upper limit of this interval and the corrected mean or the difference between the corrected mean and the lower limit of this interval is termed the *uncertainty of measurement u*. Mostly, but not always, the two differences are equal in value, see subclause 7.1.

Note. The entire span of the interval in which the true value of the measurand is presumed to lie, in other words the difference between the upper and lower limits of the interval, shall *not* be termed the uncertainty of measurement.

Below, basic rules for determining the uncertainty of measurement u are stated. The uncertainty of measurement u has two components. One of these components relates to the random errors (random component u_z) whilst the other component relates to the unknown systematic errors (systematic component u_s).

6.1 Value of the random component u_z

For determining the value of the random component u_z (see subclause 3.2) the following three cases shall be distinguished:

6.1.1 Series of measurements under repeatability conditions with unknown repeatability standard deviation σ_r

Assuming a series of measurements made under repeatability conditions (corresponding to subclause 4.1), the following applies:

$$u_z = \frac{t}{\sqrt{n}} \cdot s \quad (5)$$

As explained in detail in subclause 5.3, this is half the span of the confidence interval for the expectation μ of the series of measurements with the confidence level indicated there.

6.1.2 Series of measurements under repeatability conditions with few individual values and with known repeatability standard deviation σ_r

It often happens that each individual measurement costs a great deal, but that at the same time the standard deviation σ_r of the random errors of the method of measurement is known from earlier measurements. It is then appropriate to use

$$u_z = \frac{t_\infty}{\sqrt{n}} \cdot \sigma_r \quad (6)$$

With the few n measured values x_i the mean $\bar{x}$ is determined considerably better than with only one measured value. On the other hand, the confidence interval calculated from these few measured values (cf. subclause 6.1.1) would be significantly broader than that calculated by using the value σ_r known from experience (see example A.2 in Appendix A).

6.1.3 Individual measured value with known repeatability standard deviation σ_r

If only an individual measured value is available and if the repeatability standard deviation σ_r is known (see subclause 4.1) then the following applies:

$$u_z = t_\infty \sigma_r \quad (7)$$

If the repeatability r is stated (see subclause 4.1) it is possible to calculate from it the repeatability standard deviation $\sigma_r = r/2{,}77$.

If σ_r is not known, an expedient is to adopt the u_z value quoted by the measuring instrument manufacturer or to estimate a value for u_z from experience.

Note to 6.1. The equations given in subclause 5.3 apply subject to the proviso of normal distribution of the measured values. If this proviso does not obtain, it will usually not be possible to state a confidence interval for the expectation μ linked with a *probability statement*. In such cases, however, it is appropriate to indicate the random component u_z of the uncertainty of measurement by $s/\sqrt{n}$ where s is the (empirical) standard deviation and n the number of individual values.

6.2 Value of the systematic component u_s

Generally speaking, the *systematic component* u_s can only be estimated by applying adequate experimental experience (or reliable data from the manufacturer). When making such estimates only such figures should be used from which it can be expected that they will not be exceeded. In general, in the absence of more accurate knowledge, the magnitude of unknown positive systematic errors and the magnitude of unknown negative systematic errors will be taken as equal, and hence only a single value u_s will be stated.

Note. A possible method of arriving at a certain quantitative assessment of unknown systematic errors in specific cases is furnished by round robin tests following a defined (standard) method of measurement, performed under strict observance of the test conditions and involving a sufficiently large number of participating laboratories (see DIN ISO 5725 and DIN 51 848 Part 1 to Part 3). The evaluation of such round robin tests (variance analysis) leads to the two quantities repeatability r and reproducibility R already described in clause 4. The difference between r and R depends on systematic errors which are unknown but differ in magnitude in the various laboratories so that in this way a certain estimate of their magnitudes can be made. (Compare, however, the note concerning subclause 3.3.3 and the advantage of different methods of measurement for clarifying systematic errors).

6.3 Combining the components u_z and u_s to give the uncertainty of measurement u

There are various methods of combining the components u_z and u_s to give the uncertainty of measurement u. If the unknown systematic errors cannot be estimated, u_z has to be stated as the uncertainty of measurement, plus a note to the effect that in u only random errors are taken into account.

6.3.1 Method 1 (linear addition)

The simplest form of combining the components, and at the same time the one offering the greatest security against the risk of underestimating the uncertainty of measurement, is linear addition of the two components

$$u = u_z + u_s \quad (8)$$

The additive combination is recommendable in all cases when one of the two components is considerably larger than the other; the risk of overestimating the uncertainty of measurement is then also small.

6.3.2 Method 2 (addition of squares)

If it can be taken for granted that it is permissible to treat the systematic component u_s in the same way as the random component u_z, the following equation may be adopted:

$$u = \sqrt{u_z^2 + u_s^2} \quad (9)$$

Subject to the proviso mentioned, the square root of the sum of the squares of u_z and u_s is always recommendable if the magnitudes of the two components u_z and u_s are approximately equal.

Note. In cases of doubt it shall be stated whether method 1 or method 2 has been used.

7 Result of measurement

7.1 General information concerning the result of measurement

The mean $\bar{x}$ of a series of measurements (see subclause 5.1) has to be corrected for the known systematic errors (see subclause 3.3.2). By way of correction K (see subclause 8.2.5) the corrected mean $\bar{x}_E$ is obtained:

$$\bar{x}_E = \bar{x} + K \quad (10)$$

Apart from the mean $\bar{x}_E$ to which correction K has been applied, the *result of measurement* y must always contain the uncertainty of measurement u as indicated in subclause 6.3:

$$y = \bar{x}_E \pm u \quad (11)$$

In equation (11) it is assumed that, as is usually the case, the uncertainty of measurement is the same in both the upward and downward directions (in respect of the mean). For the case when the uncertainty of measurement upward and downward is unequal it will as a rule be necessary to state the two uncertainties of measurement separately in addition to the mean $\bar{x}_E$. If only *one* value shall be stated for the uncertainty of measurement, the larger value shall be taken.

It is often desirable for the standard deviation also to be quoted.

Note 1. In the case of an individual value the mean and the corrected mean are replaced by the individual values x and x_E.

Note 2. For results of measurement it is not permitted for a certain "accuracy" to be stated *quantitatively*; only the term *uncertainty of measurement* (see clause 6) shall be used.

Examples for stating the result of measurement:
Here, as in industrial practice in general, $1 - \alpha = 95\,\%$ shall be taken as the confidence level.

a) The random component of the uncertainty of measurement in the indication of a thermometer graduated in $^1/_{10}$ °C is $u_z = 0{,}02$ °C. With the systematic component u_s negligible in this case, the result of measurement is $t = (21{,}54 \pm 0{,}02)$ °C.

b) The relative uncertainty of measurement (see subclause 7.2) in the determination of the thermal conductivity of metals is $\varepsilon = 2\,\%$.
Result of measurement (for an Al sample):

$$\lambda = \bar{x}_E\,(1 \pm \varepsilon) = 220{,}0\,(1 \pm 0{,}02)\ \mathrm{W \cdot K^{-1} \cdot m^{-1}}$$

c) The frequency measurement $f = 10{,}38062$ MHz was uncertain by 10 Hz.
Result of measurement:
$f = 10{,}38062\ \mathrm{MHz} \pm 1 \cdot 10^{-5}\ \mathrm{MHz}$

d) In a single measurement using an Ubbelohde viscometer the kinematic viscosity of a mineral oil sample was found as $\nu = 125{,}0\ \mathrm{mm^2 \cdot s^{-1}}$. The reproducibility standard deviation for this known from long experience is $\sigma_R = 0{,}3\ \mathrm{mm^2 \cdot s^{-1}}$. In this case the result of measurement reads as follows:

$$\nu = x_E \pm t_\infty \sigma_R = 125{,}0\,\mathrm{mm^2 \cdot s^{-1}} \pm 1{,}96 \cdot 0{,}3\,\mathrm{mm^2 \cdot s^{-1}}$$
$$\nu = (125{,}0 \pm 0{,}6)\,\mathrm{mm^2 \cdot s^{-1}}.$$

7.2 Relative uncertainty of measurement

The relative uncertainty of measurement ε is the quotient obtained by dividing the uncertainty of measurement u by the corrected mean $\bar{x}_E$:

$$\varepsilon = \frac{u}{\bar{x}_E} \qquad (12)$$

In this case the result of measurement for the symmetrical condition reads as follows:

$$y = \bar{x}_E\,(1 \pm \varepsilon) \qquad (13)$$

7.3 Indicating the result of measurement for very accurate measurements

The result of measurement from a series of measurements with n independent individual measured values is stated completely for very accurate measurements if it contains the following data:

- corrected mean $\bar{x}_E$ (mean given by equation (1) corrected for the known systematic errors)
- number n of individual measured values and standard deviation s
- random component u_z for chosen $(1 - \alpha)$
- systematic component(s) u_s; if important, the proportions contained in u_s shall also be stated.

8 Assessment of measuring instruments and measuring equipment

Measuring instruments and measuring equipment (hereafter briefly referred to as measuring equipment) can be assessed in terms of the systematic and random errors of the measured values obtained with them.

8.1 Assessment of measuring equipment in terms of random errors

The random errors (see subclause 3.2) of the measured values obtained with a measuring equiment are assessed with the aid of a series of measurements under repeatability conditions. The evaluation yields a quantitative statement concerning the *precision* of the measuring equipment, for example in the form of the repeatability standard deviation (see subclause 4.1).

Note. It shall be borne in mind that the repeatability standard deviation may alter with any variation in the measurement conditions (other values of the measurand, measurement in a different measuring range).

8.2 Assessment of measuring equipment in terms of systematic errors

8.2.1 General

The difference between the expectation μ and the true value x_w cannot be determined exactly because both the expectation μ and the true value x_w are unknown in principle. Instead of the expectation μ use is made of the arithmetic mean $\bar{x}_a$ of an adequately large number of measured values, and instead of the true value x_w the conventional true value x_r is taken. The conventional true value x_r is determined with a measuring equipment whose unknown systematic error is considerably smaller than that of the measuring equipment to be assessed. It is often determined with a standard instrument or a standard.

Note. Hence, instead of the systematic error to be assessed

$$\mu - x_w$$

only its estimate

$$\bar{x}_a - x_r$$

is known, whilst the difference

$$\mu - x_w - (\bar{x}_a - x_r) = (\mu - \bar{x}_a) - (x_w - x_r)$$

continues to persist as the unknown systematic error. It does not need to persist, however; for example $\mu - \bar{x}_a$ can be reduced by increasing the series of measurements under repeatability conditions and $x_w - x_r$ by having recourse to a still more accurate measuring equipment for checking the measuring equipment to be assessed.

8.2.2 Indicating measuring instrument (measuring equipment)

If the measuring equipment to be tested is an *indicating measuring instrument*, then $\bar{x}_a$ is the arithmetic mean of the "indicated" values (subscript a) of the measurand read off on this measuring instrument (often termed "indications"). If it is known that the random errors of the measuring instrument to be assessed are considerably smaller than its systematic errors, it is sufficient each time to determine only one indication (output) x_a. Hence the *ascertained systematic error* is

$$A_a = \bar{x}_a - x_r \quad \text{or} \quad A_a = x_a - x_r \qquad (14)$$

Note 1. The indication x_a in this sense is also termed the actual indication in some branches of metrology.

The prefixes "actual" and "desired" should, however, be avoided in this context, since their use often leads to misunderstanding.

Note 2. It is expedient to reach agreement case by case as to the manner in which the indication is to be determined (e. g. as an individual measured value or as the mean of a given number of individual values, e. g. $n = 3$).

Note 3. Measuring instruments with indirect output (see DIN 1319 Part 2) are to be assessed in the same way. Instead of the indicated values (values read off) the output values (measurement signals or other representations) shall be used.

8.2.3 Material measure

If the measuring equipment to be tested is a *material measure*, its indication (output) x_a (see subclause 8.2.2) is in correspondence with the inscribed value x_A indicated on the material measure by the inscription (subscript A). The conventional true value x_r is determined by measurement of the material measure, for example by comparison with a standard. Hence the *ascertained systematic error* (of the inscribed value) is

$$A_A = x_A - x_r \tag{15}$$

Note 1. In this standard a material measure is considered a measuring instrument. For its assessment, therefore, the inscription (indication) is tested and the systematic error $x_A - x_r = A_A$ by which the inscribed value deviates from the conventional true value is ascertained and stated.

If, on the other hand, as in certain areas of linear measurement, e. g. in the case of gauge blocks, a material measure is considered an object to be measured (workpiece), the test is concerned with determining how far the conventional true value deviates from the inscribed value (nominal size). It is then usual to state the error as the difference $x_r - x_A$; this error has the opposite sign to the systematic error A_A ascertained by means of equation (15).

Note 2. The value indicated by the inscription on the material measure is also termed the "nominal value (nominal size)". This shall be avoided in view of the risk of confusion with the concept of nominal value widely used with another meaning in technology (see, for example, DIN 55 350 Part 12 and DIN 40 200). In cases of doubt, the basis to which the ascertained error relates (e.g. indication, inscription) shall always be stated.

8.2.4 Relative indication of the ascertained systematic error

For the relative indication various reference values are possible. In most cases the conventional true value is used as the reference value. In the case of indicating measuring equipment the systematic error is related in the majority of instances to the end value of the measuring range (see, for example, DIN 43 780). The relative values of the ascertained systematic error are usually stated in percent. The reference value applied must be made clear by means of a formula or by the text.

8.2.5 Correction

The *correction* K has the same magnitude as the ascertained systematic error as given by equation (14) and equation (15), but has opposite sign, hence

$$K = -A_a \tag{16}$$

and for material measures

$$K = -A_A \tag{17}$$

8.2.6 Examples

8.2.6.1 Indicating measuring instrument: liquid-filled thermometer, scale interval 0,1 °C

Mean of $n = 10$ indications in the repeatability measurement series:	$\bar{t}_a = 20{,}15$ °C
Conventional true value of temperature, obtained as indication of a standard thermometer, e. g. a platinum resistance thermometer:	$t_r = 20{,}01$ °C
Ascertained systematic error of indication of the thermometer:	$A_A = \bar{t}_a - t_r = 0{,}14$ °C
Correction:	$K = t_r - \bar{t}_a = -0{,}14$ °C

8.2.6.2 Indicating measuring instrument: voltmeter, measuring range 3 V, accuracy class 0,05 as specified in DIN 43 780

Mean of $n = 10$ indications in the repeatability measurement series:	$\bar{U}_a = 1{,}8249$ V
Conventional true value of voltage, determined with the aid of a voltage calibrator of high accuracy:	$U_r = 1{,}8240$ V
Ascertained systematic error of indication of the measuring instrument:	$R_a = \bar{U}_a - U_r = 0{,}0009$ V
Relative value of the ascertained systematic error referred to the end value U_e:	$\frac{\bar{U}_a - U_r}{U_e} = \frac{0{,}0009}{3} = 0{,}0003 = 0{,}03$ %
Correction:	$K = -0{,}0009$ V

8.2.6.3 Material measure: 1 Ω resistor

Resistance as per inscription:	$R_A = 1{,}0000\ \Omega$
Conventional true value:	$R_r = 1{,}0019\ \Omega$
Ascertained systematic error of the inscribed value of the material measure:	$A_A = R_A - R_r = -0{,}0019\ \Omega$
Relative indication referred to the conventional true value:	$(R_A - R_r)/R_r = -0{,}19$ %
Correction:	$K = R_r - R_A = 0{,}0019\ \Omega$

8.2.6.4 Mechanical material measure: 70 mm gauge block

a) Considered as measuring instrument

Length as per inscription:	$L_A = 70$ mm
Conventional true value:	$L_r = 69{,}998$ mm

Ascertained systematic error of the inscribed value of the gauge block:	$A_A = L_A - L_r = 0{,}002$ mm
Relative indication referred to the conventional true value:	$(L_A - L_r)/L_r = 0{,}0029$ %
Correction:	$K = -A_A = -0{,}002$ mm

b) Considered as an object to be measured (see subclause 8.2.3, Note 1)

Length as per inscription:	$L_A = 70$ mm
Conventional true value:	$L_r = 69{,}998$ mm
Deviation of conventional true value from inscribed value:	$L_r - L_A = -0{,}002$ mm

8.3 Limits of error

8.3.1 Concept

Limits of error as agreed maximum magnitudes of (positive or negative) errors of indication (output) of measuring equipment (measuring instruments).

Limits of error are specified principally in respect of systematic errors of the measured values from the conventional true value or some other defined or agreed value of the measurand; they shall also not be exceeded by random errors (see Appendix A, subclause A.4).

Different limits of error may be specified for the magnitudes of the positive errors and negative errors. These are termed the upper limit of error G_0 and the lower limit of error G_u. In the majority of cases, however, the upper and lower limits of error are the same. They are then referred to as symmetrical limits of error and designated by G.

Hence, in the symmetrical case, the following basic relationship exists for the indication (output) x_a of a measuring equipment (a measuring instrument) in respect of the limit of error G:

$$x_r - G \leq x_a \leq x_r + G \qquad (18)$$

In the unsymmetrical case the following applies:

$$x_r - G_u \leq x_a \leq x_r + G_o \qquad (19)$$

In the case of material measures, x_A shall be used in equation (18) and equation (19) instead of x_a.

Limits of error may be stated in units of the quantity concerned or referred to the end value of the measuring range or referred to some other value. The relative statement is usually made in percent, for example in percent of the end value of the measuring range of an electrical measuring instrument.

Note 1. The *limits of error* G indicate within what limits a measured value (result of measurement) deviates from the conventional true value, i. e. may be *incorrect*, and they are dictated primarily by systematic errors deriving mostly from the unavoidable discrepancies arising during manufacture of the measuring instruments.

The range fixed by the limits of error shall be considerably larger than the random component u_z of the uncertainty of measurement of an individual value (see subclause 6.1.2).

Limits of error comprise – unless specially agreed otherwise – the ascertained systematic errors as well as discrepancies dictated by the technical possibilities and unavoidable inconsistencies of manufacture of measuring equipment, and by ageing effects. If the limits of error apply only under specific (qualifying) secondary conditions (e. g. at a temperature of 20 °C or in a temperature range of 10 to 30 °C), these secondary conditions shall be stated.

Note 2. In the case of many measuring instruments the non-transgression of the limits of error is only guaranteed subject to observance of specific reference conditions for the prevailing influence quantities $A, B, \ldots$. These influence quantities are physical quantitites which are not an object of the measurement (e. g. ambient temperature, humidity, atmospheric pressure, interference fields). They do, however, exert an unwanted influence – from outside – on the indication (output) of the measuring instruments and hence on the measured value of the measurand and in this way bring about systematic errors. For details regarding electrical measuring instruments, see DIN 43 780.

8.3.2 Fixing the limits of error

Limits of error are fixed by agreements or specifications, for example in the Ordinance on weights and measures, in standards (for example, standards of the 57 472/VDE 0472 series, DIN 43 780) and in other regulations, as well as contractually.

8.3.2.1 Verification limits of error

Verification limits of error are limits of error prescribed by legislation in the Ordinance on weights and measures.

Note. A measuring equipment is given the verification mark by the verification authority only if no deviations of the measured values from the conventional true value of the more accurate measuring equipment used for testing or of the standard or standard procedure adopted for testing are ascertained whose magnitudes are greater than the verification limit of error. If errors of measurement of larger magnitude are ascertained, the measuring equipment being tested counts as faulty within the meaning of quality control and cannot be given the verification mark.

8.3.2.2 In-service limits of error

In-service limits of error are the limits of error applicable to the measuring instruments in practical use. They are prescribed by legislation.

Note. The in-service limits of error are generally symmetrical and amount to twice the verification limits of error.

8.3.3 Stating limits of error

Limits of error are magnitudes and are therefore stated without sign, i.e. as the value G in the symmetrical case and as the values G_u and G_o in the unsymmetrical case.

Note. The earlier practice of stating limits of error with the sign ± in the symmetrical case and with the – sign and the + sign in the unsymmetrical case is not recommended. It would then no longer be permitted to equate the noncompliance with the requirement given by a limit of error by the words "transgression of the limit of error"; a transgression of the lower limit of error (e. g. a deviation of − 4 when the limit of error is − 5) would mean that the requirement given by this limit of error was just complied with.

The following cases can arise when stating limits of error.

8.3.3.1 Symmetrical limits of error G

The normal case in practical metrology is for the limits of error to be symmetrical. For these only one value $G = G_u = G_o$ is stated. Hence, conversely, the stating of only *one* value G signifies that symmetrical limits of error are involved.

8.3.3.2 Unsymmetrical limits of error G_u and G_o

In these cases, which occur less frequently, the two limits of error G_o and G_u are to be stated separately.

8.3.3.3 Indirect stating of the limits of error by giving the limiting values for the indication (output)

Instead of the limits of error in respect of a conventional true value x_r of the measurand, a lower and an upper limiting value (minimum value and maximum value) are stated for the indication of the measuring equipment. For example, see subclause 8.3.3.4.5.

8.3.3.4 Examples for stating limits of error

8.3.3.4.1 Stating the limits of error in units of the measurand, illustrated by the example of the verification limit of error of a mercury-in-glass thermometer:

$G = 0{,}2\,°C$ at $20{,}0\,°C$

or $G = 0{,}2$ K at $20{,}0\,°C$

The indicated temperature value t_a may therefore lie in the following range about the verification value $t_r = 20{,}0\,°C$:

$$20{,}0\,°C - 0{,}2\,°C \leq t_a \leq 20{,}0\,°C + 0{,}2\,°C$$

8.3.3.4.2 Stating unsymmetrical limits of error in units of the measurand, illustrated by the example of the verification limits of error of clinical thermometers:

$$G_u = 0{,}15\,°C; \qquad G_o = 0{,}10\,°C$$

For the indicated temperature value t_a, therefore, the following condition applies:

$$t_r - 0{,}15\,°C \leq t_a \leq t_r + 0{,}10\,°C$$

8.3.3.4.3 Stating a unilateral limit of error in units of the measurand, illustrated by the example of the verification limits of error of a material measure:

Weight conforming to the medium class of error limits, inscription 500 g; conventional true value of the weight m_r:

$$G_u = 100\,\text{mg}; \qquad G_o = 0\,\text{mg}$$

Hence the inscribed value $m_A = 500$ g is to lie within the following limits:

$$m_r - 100\,\text{mg} \leq 500\,\text{g} \leq m_r + 0\,\text{mg}$$

This is equivalent to the statement that the verified weight bearing the inscription 500 g shall not be lighter than 500 g and not more than 100 mg heavier than 500 g.

8.3.3.4.4 Stating the relative value of the limit of error referred to the end value, illustrated by the example of the limit of error for voltage measuring equipment of high accuracy with the end value U_e:

$$\frac{G}{U_e} = 0{,}1\,\%$$

Hence, the following applies for an indication U_a of the voltmeter in respect of the conventional true value U_r of the voltage:

$$U_r - 0{,}1\,\%\,U_e \leq U_a \leq U_r + 0{,}1\,\%\,U_e$$

8.3.3.4.5 Example (thermometer) for indirectly stating the limits of error: for a defined conventional true value the limits for the indication (output) are specified:

- conventional true value 20,0 °C
- minimum value 19,8 °C
- maximum value 20,2 °C

The indicated temperature value t_a may therefore lie in the range

$$19{,}8\,°C \leq t_a \leq 20{,}2\,°C$$

if the conventional true value is 20,0 °C.

Appendix A

Examples

A.1 Measurement of a base quantity

A.1.1 The intention is to measure the length of a rod having a value of 150 mm (inscription) with an indicating length measuring instrument (visual method). Testing of the length measuring instrument has revealed that its indication in the measuring range concerned has a correction $K = +0{,}06$ mm (see subclause 8.2.5). For measuring the length of the rod 20 individual measurements were made under repeatability conditions. These $n = 20$ individual values are listed below under A.1.2. From them the arithmetic mean $\bar{x}$, the (empirical) standard deviation s and the confidence limits for the expectation for a confidence level $1 - \alpha = 95$ % shall be calculated.

To make the details of the evaluation method clear, the calculation procedure is presented in detail with the aid of this simple example; generally speaking, the computer will operate with the second form of equation (2) in subclause 5.2 (however, an adequate number of places shall be ensured when using the computer for this calculation).

A.1.2 Individual values x_1 to x_{20}

Measurement No.	Length x_i in mm	$10^2 \cdot (x_i - \bar{x})$ in mm	$10^4 \cdot (x_i - \bar{x})^2$ in mm²
1	150,14	+12	144
2	150,04	+ 2	4
3	149,97	− 5	25
4	150,08	+ 6	36
5	149,93	− 9	81
6	149,99	− 3	9
7	150,13	+11	121
8	150,09	+ 7	49
9	149,89	−13	169
10	150,01	− 1	1
11	149,99	− 3	9
12	150,04	+ 2	4
13	150,02	0	0
14	149,94	− 8	64
15	150,19	+17	289
16	149,93	− 9	81
17	150,09	+ 7	49
18	149,83	−19	361
19	150,03	+ 1	1
20	150,07	+ 5	25
$\sum$	3000,40	0	1522

A.1.3 Mean

$$\bar{x} = \frac{1}{20} \sum_{i=1}^{20} x_i = \frac{1}{20} \cdot 3000{,}40\,\text{mm} = 150{,}02\,\text{mm}$$

This mean $\bar{x}$ has to be corrected by the ascertained correction K.

The corrected mean is

$$\bar{x}_E = \bar{x} + K = 150{,}02\,\text{mm} + 0{,}06\,\text{mm} = 150{,}08\,\text{mm}$$

A.1.4 (Empirical) standard deviation s

The standard deviation s is calculated by means of equation (2).

$$s = \sqrt{\frac{1}{19} \sum_{i=1}^{20} (x_i - 150{,}02\,\text{mm})^2}$$

$$= \sqrt{\frac{1}{19} \cdot 1522 \cdot 10^{-4}\,\text{mm}^2} = 0{,}09\,\text{mm}$$

A.1.5 Confidence limits for the expectation

$1 - \alpha = 95$ % was given as the confidence level. The values $t = 2{,}09$ and $t/\sqrt{n} = 0{,}48$ are found for this from table 1 in clause 5 for $n = 20$. Hence the following are obtained for the confidence limits of the mean (equation (4)):

lower confidence limit

$$\bar{x}_E - \frac{t}{\sqrt{n}} s = 150{,}08\,\text{mm} - 0{,}04\,\text{mm} = 150{,}04\,\text{mm}$$

upper confidence limit

$$\bar{x}_E + \frac{t}{\sqrt{n}} s = 150{,}08\,\text{mm} + 0{,}04\,\text{mm} = 150{,}12\,\text{mm}$$

A.1.6 Result of measurement

The result of measurement for the wanted length L of the rod is (equation (10)):

$$L = \bar{x}_E \pm u$$

From the equations (8) and (9) it is found that u has a random component u_z (equations (5) to (7)) and a systematic component u_s. In the present case $u_z = 0{,}04$ mm (see subclause A.1.5). The systematic component u_s in the present case is so small that it may be ignored. Hence the confidence interval for the expectation becomes the confidence interval for the true value.

The following applies:

Length $L = 150{,}08$ mm $\pm$ 0,04 mm

The result of measurement can also be stated using the relative uncertainty of measurement ε (see equation (11)). In this case

$$\varepsilon = \frac{u}{\bar{x}} = \frac{0{,}04}{150{,}08} = 2{,}7 \cdot 10^{-4}$$

The rounded result of measurement is now:

$$L = 150{,}08 \cdot (1 \pm 0{,}0003)\,\text{mm} = 150{,}08 \cdot (1 \pm 0{,}03\,\%)\,\text{mm}$$

A.2 Example of a derived quantity

It is intended to measure the thermal conductivity λ of a sample of structural steel at 203 °C. As a suitable method it was decided to measure the axial temperature gradient in a cylinder with a length of 90 mm and a diameter of 50 mm conveying the heat flux to be measured.

In a repeatability series of measurements $n = 5$ individual values were measured and in this way the mean $\bar{x} = 54{,}3\ \text{W} \cdot \text{K}^{-1} \cdot \text{m}^{-1}$ was obtained. This mean has to be corrected for a *known* systematic error brought about mainly by unavoidable, but computable heat losses and by measured distortions of the temperature field. These errors lead to a correction $K = +0{,}4\ \mathrm{W \cdot K^{-1} \cdot m^{-1}}$. The corrected mean is

$$\bar{x}_E = \bar{x} + K = (54{,}3 + 0{,}4)\ \mathrm{W \cdot K^{-1} \cdot m^{-1}} = 54{,}7\ \mathrm{W \cdot K^{-1} \cdot m^{-1}}$$

From numerous earlier tests the standard deviation $\sigma_r = 0{,}34\ \mathrm{W \cdot K^{-1} \cdot m^{-1}}$ is known. In this way the random component u_z of the uncertainty of measurement is found (with $1 = \alpha = 95\ \%$; $\sqrt{n} = \sqrt{5} = 2{,}24$; $1{,}96/\sqrt{n} = 0{,}88$ from table 2):

$$u_z = 0{,}88 \cdot 0{,}34\ \mathrm{W \cdot K^{-1} \cdot m^{-1}} = 0{,}3\ \mathrm{W \cdot K^{-1} \cdot m^{-1}}$$

Also linked to such measurements of thermal conductivity is an *unknown* systematic error u_s; this is brought about by unknown heat losses, perturbations due to mounting, and non-measurable or non-calculable distortions of the temperature field. From long experience u_s is estimated at the following value symmetrical on either side of the mean

$$u_s = 0{,}9\ \mathrm{W \cdot K^{-1} \cdot m^{-1}}$$

From this the uncertainty of measurement is found from equation (8) to be

$$u = u_z + u_s = (0{,}3 + 0{,}9)\ \mathrm{W \cdot K^{-1} \cdot m^{-1}} = 1{,}2\ \mathrm{W \cdot K^{-1} \cdot m^{-1}}.$$

Therefore the result of measurement is:

Thermal conductivity $\lambda = (54{,}7 \pm 1{,}2)\ \mathrm{W \cdot K^{-1} \cdot m^{-1}}$

Taking the relative uncertainty of measurement as $\varepsilon = u/\lambda = 0{,}022$, the result of measurement is written as follows:

Thermal conductivity $\lambda = 54{,}7 \cdot (1 \pm 0{,}022)\ \mathrm{W \cdot K^{-1} \cdot m^{-1}}$, $= 54{,}7 \cdot (1 \pm 2{,}2\ \%)\ \mathrm{W \cdot K^{-1} \cdot m^{-1}}$

A.3 Estimating an unknown systematic error

When measuring the length of measuring rules (levelling staffs) with the aid of a comparator the value obtained under repeatability conditions (without change of position of the rules) for the random component u_z was 5 µm for the confidence level $1 - \alpha = 95\ \%$, the number of measurements n being 10. Further measurements with the rule in different positions (change of mounting) led to different measured values with deviations from the mean of the repeatability series of measurements ranging from -30 µm to $+30$ µm. It can be concluded from this that in this case unknown systematic disturbing influences are effective from the start. They have to be allowed for by a systematic component u_s of the uncertainty of measurement and, in view of the random component $u_z = 5$ µm, are estimated as $u_s \approx 25$ µm.

A.4 Meaning of limits of error

The concepts defined in subclause 8.3 are illustrated by the simple example below (see also DIN 1319 Part 2).

A mercury thermometer has a measuring range from $-10\ ^\circ\mathrm{C}$ to $110\ ^\circ\mathrm{C}$ and is divided in $^1/_{10}\ ^\circ\mathrm{C}$ (scale interval $0{,}1\ ^\circ\mathrm{C}$). As specified in Appendix 14 of the Ordinance on weights and measures the symmetrical verification limits of error (see subclause 8.3.2.1) for this thermometer are $G = 0{,}2$ K or $0{,}2\ ^\circ\mathrm{C}$.

The thermometer is tested in a water bath at the "conventional true" temperature of $20{,}00\ ^\circ\mathrm{C}$ as determined with the aid of a standard thermometer and then shows the temperature $t_a = 20{,}12\ ^\circ\mathrm{C}$ (indication, reading). At this measuring point, therefore, the ascertained systematic error A_a is:

Indication $-$ conventional true value $= 20{,}12\ ^\circ\mathrm{C} - 20{,}00\ ^\circ\mathrm{C} = +0{,}12\ ^\circ\mathrm{C}$. The correction K is therefore $K = -0{,}12\ ^\circ\mathrm{C}$.

These values are ascertainable with sufficient reliability, since the random component of the uncertainty of measurement of an individual value as determined for this instrument is only $u_z = 0{,}02\ ^\circ\mathrm{C}$ (the unknown systematic component u_s may be ignored in this case). The user of this measuring instrument may now either content himself with the fact that his thermometer has received the verification mark and indicates "correctly" within the limits $t_r - 0{,}2\ ^\circ\mathrm{C}$ to $t_r + 0{,}2\ ^\circ\mathrm{C}$, or he will evaluate more precisely and in this case has to regard the difference between an indication – in the region of $20\ ^\circ\mathrm{C}$ e. g. $t_a = 21{,}43\ ^\circ\mathrm{C}$ in the context of this example – and the conventional true value as a correction as per subclause 8.2.5:

$$t_r = t_a + K = 21{,}43\ ^\circ\mathrm{C} - 0{,}12\ ^\circ\mathrm{C} = 21{,}31\ ^\circ\mathrm{C}$$

In this case a measured value determined with his instrument (with confidence level $1 - \alpha = 95\ \%$) is only "uncertain" by $u_z = 0{,}02\ ^\circ\mathrm{C}$.

With the aid of the example of the thermometer chosen here, figure 1 makes it clear how limits of error are to be interpreted, the conventional true temperature $t_r = 20{,}00\ ^\circ\mathrm{C}$ being selected as the reference value for the sake of simplicity.

Case 1 corresponds to a thermometer with indication $t_{a1} = 20{,}12\ ^\circ\mathrm{C}$; t_{a1} thus lies within the limits $t_r - G$ and $t_r + G$: the verification limit of error G is not exceeded. This thermometer receives a verification mark.

Case 2 represents a borderline case in which the indication of another thermometer $t_{a2} = 19{,}80\ ^\circ\mathrm{C}$ is equal to the limit $t_r - G$: the verification limit of error G is only just not transgressed. This thermometer is given a verification mark irrespective of the fact that the random component of the uncertainty of measurement u_z, which is assigned to the indication t_{a2}, at $u_z = 0{,}02$ could also lead to values lying outside the limits $t_r - G$ and $t_r + G$.

In case 3 the indication given by a third thermometer $t_{a3} = 20{,}21\ ^\circ\mathrm{C}$ is outside the limits $t_r - G$ and $t_r + G$: The verification limit of error G is transgressed by the indication t_{a3}. This thermometer does not receive a verification mark, regardless of the fact that the random component of the uncertainty of measurement u_z, which is assigned to the indication t_{a3}, at $u_z = 0{,}02\ ^\circ\mathrm{C}$ could also lead to values lying within the limits $t_r - G$ and $t_r + G$.

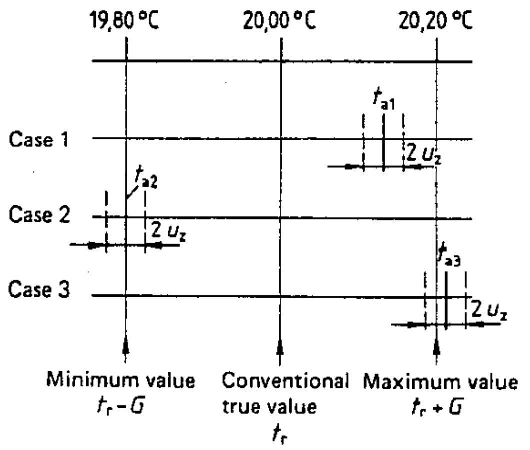

Figure 1. Different cases of the indications t_{a1}, t_{a2}, t_{a3}

Standards referred to and other documents

DIN 1313	Physical quantities and equations; terminology, methods of writing
DIN 1319 Part 1	Basic concepts in metrology; measuring, counting, testing
DIN 1319 Part 2	Basic concepts in metrology; terminology relating to the use of measuring instruments
DIN 40 200	Nominal value, limiting value, rated value, rating; terminology
DIN 43 780	Electrical measuring instruments; direct-acting indicating measuring instruments and accessories
DIN 51 848 Part 1	Testing of mineral oils; precision of test methods; general, terminology and its application to mineral oil standards containing requirements
DIN 51 848 Part 2	Testing of mineral oils; test errors, planning of round robin tests
DIN 51 848 Part 3	Testing of mineral oils; test errors, calculation of test errors
DIN 55 302 Part 1	Statistical evaluation methods; frequency distribution, mean and variability, basic concepts and general calculation methods
DIN 55 302 Part 2	Statistical evaluation methods; frequency distribution, mean and variability, calculation methods for special cases
DIN 55 350 Part 12	Concepts in quality assurance and statistics; concepts in quality assurance, concepts relating to characteristics
DIN 55 350 Part 13	Concepts in quality assurance and statistics; concepts in quality assurance, concepts relating to accuracy
Standards of the DIN 57 472/VDE 0472 series	Testing of cables and insulated conductors
DIN ISO 5725	Precision of test methods; determination of repeatability and reproducibility by inter-laboratory tests
ISO 3534	Statistics; vocabulary and symbols
Ordinance on weights and measures, Appendix 14	Temperature measuring instruments

Other relevant standards

DIN 55 350 Part 11 Concepts in quality assurance and statistics; concepts in quality assurance, basic concepts

Previous editions

DIN 1319: 07.42, 01.62, 12.63; DIN 1319 Part 3: 12.68, 01.72

Amendments

Completely revised compared with the January 1972 edition; assessment of series of measurements and measuring instruments separated; clause on error propagation deleted. See Explanatory notes.

Explanatory notes

The previous edition of DIN 1319 Part 3 has undergone fundamental revision in this standard. Detailed discussions on this revised standard have been held with experts outside *AEF* and have been conducted in particular with the Committee *Qualitätssicherung und angewandte Statistik (AQS)* of *DIN.* Part 3 deals with two topics which need to be differentiated from one another but are applied jointly by the user in practice and therefore belong together:

a) determining the value of a physical quantity (measurand) from a series of measurements with n individual values, clauses 1 to 7.

b) assessing measuring instruments with the aid of the concepts "ascertained systematic error" and "correction", clause 8.

The earlier Error propagation clause has been deleted. Its content is to be dealt with in revised form in a further part of DIN 1319. Instrumental for this decision was the fact that proposals for a general and unified treatment for combining the uncertainty of measurement u from two components u_z (random type) and u_s (systematic type) has been made from various sides and are still being examined (cf. S. Wagner, *On the Quantitative Characterization of the Uncertainty of Experimental Results in Metrology. PTB-Mitt.* 89 (1979) No 2, p. 83: also VDE/VDI-Richtlinie (Code of Practice) 2620 Part 1).

International Patent Classification

G 01

Alphabetical index

The figures following the key words are clause/subclause numbers