

**General Definitions
and Specification Factors
for Gears, Gear Pairs and Gear Trains**

**DIN
868**

Allgemeine Begriffe und Bestimmungsgrößen für Zahnräder, Zahnradpaare und Zahnradgetriebe

This Standard contains the general definitions and symbols for gears, gear pairs and gear trains as well as the definitions of their parameters. The content of this Standard has been brought into line with the recommendation ISO/R 1122 – 1969 issued by the International Organization for Standardization (ISO); for this purpose the Swiss Standard VSM 15 522 – 1974 has also been used as a source, so that extensive agreement with international standardization has been achieved. The quantities contained in ISO/R 1122 – 1969 have been supplemented in this Standard by further terms and parameters which have appeared important in connection with gear tooth geometry. The symbols used agree with DIN 3999.

Specific terms relating to particular gear tooth systems are dealt with in separate Standards.

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1 Symbols and designations

The following symbols and designations are used in this Standard:

a	Centre distance
b	Facewidth
b_C	Width of crowning
c	Bottom clearance
d	Datum surface diameter
d_N	Effective diameter
i	Transmission ratio
j	Backlash
j_e	Entry clearance
j_n	Normal backlash
j_r	Radial backlash
j_t	Circumferential backlash
j_x	Axial play
k	Number of teeth or pitch number
m	Module
n	Speed (rotational frequency)
p	Pitch
p_z	Lead
r	Datum surface radius
u	Gear ratio
v_g	Velocity of sliding
z	Number of teeth
C	Pitch point
C	End relief
C_a	Tip relief
C_b	Tooth trace relief
C_t	Root relief
C_h	Profile modification
S	Helix point
W	Working point
β	Helix angle
δ	reference cone half angle
τ	Angular pitch
ω	Angular velocity
Σ	Shaft angle

The above symbols may be supplemented by the following subscripts or auxiliary symbols:

a	for the driving gear or for parameters at the tip circle
b	for the driven gear
f	for parameters at the root surface
m	for a mean value
n	for parameters in the normal section
s	for parameters at the helix point
t	for parameters in the transverse section
w	for parameters at the working point
x	for parameters in the axial section
C	for parameters referred to end relief
red	for reduced parameters
rel	for relative parameters
1	for parameters relating to the smaller gear of a gear pair
2	for parameters relating to the larger gear of a gear pair
—	for scalar parameters
→	for vector parameters

For further symbols relating to gear teeth see DIN 3999.

Symbols are also dealt with in the following Standards:
 DIN 1302 Mathematical signs and symbols,
 DIN 1304 General symbols for use in formulae,
 DIN 1313 Method of writing physical equations in the natural sciences and technology.

2 General terms relating to a gear

2.1 Tooth

A tooth is an element projecting from a gear body and possessing a shape permitting the transmission of force and motion to the teeth of a mating gear (see Section 3.1.1).

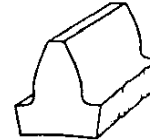


Figure 1. Tooth

2.2 Gear

A gear is a mechanical element rotatable about an axis and consisting of the gear body with its bearing surfaces and the teeth projecting from the gear body.

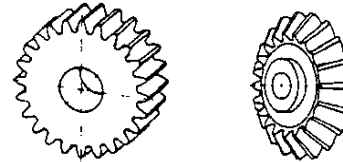


Figure 2. Gear

2.3 Gear tooth system

The tooth system of a gear comprises the totality of its teeth.

2.3.1 Tooth 1, tooth 2 etc., tooth k

For designating individual teeth, tooth 1, tooth 2 etc. are to be so marked on a transverse surface of the gear tooth system (see Section 2.8.1) that the teeth are numbered in ascending order in the counting direction. A tooth is generally designated by the letter symbol k . The next higher tooth in the counting direction is then designated by $k + 1$ and the next lower tooth in the counting direction by $k - 1$, see Fig. 3.

In the case of gear pairs with parallel axes (see Section 3.5) the transverse surfaces used for designating the teeth shall be visible from the same viewing direction; normally this is the direction in which the force is imparted to the driving gear, or some other specific, assumed direction.

In the case of gear pairs with intersecting axes (see Section 3.5) the viewing direction for both gears is generally towards the point at which the axes intersect.

In the case of gear pairs with non-parallel non-intersecting axes (see Section 3.5) the transverse surfaces generally used for designating the teeth are those which are visible from the drive side in the case of the driving gear and from the driven side in the case of the driven gear.

2.3.2 Tooth spaces

The tooth spaces are the intervals between the teeth into which the teeth of the mating gear (see Section 3.1.1) enter during the rotary motion.

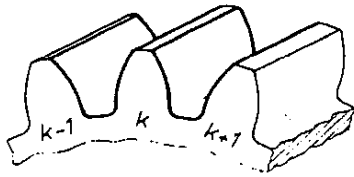


Figure 3. Designation of teeth

2.3.3 Pitch p ; right pitch, left pitch; designation

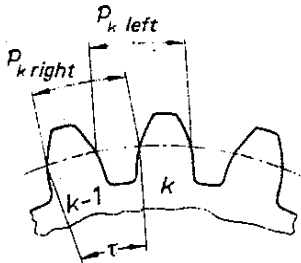
A pitch p (single pitch) is the datum surface arc (see Section 2.6) in a given section of the gear tooth system (see Section 7.3) between the corresponding flanks (see Section 7.5.3) of two adjacent teeth.

The pitch between two right flanks (see Section 7.5.2) is termed a right pitch and that between two left flanks (see Section 7.5.2) a left pitch.

The pitches p_k are the pitches (right pitch and left pitch) between tooth k and tooth $k - 1$, see Fig. 4.

2.3.4 Angular pitch τ

The angular pitch τ is the centre angle belonging to a pitch p .

Figure 4. Pitch p_k , angular pitch τ

2.3.5 Module m

The module m is the basic parameter for linear dimensions of gear tooth systems. It is found as the result of dividing the pitch p by the number π . It is stated in mm and is determined by the dimensions of the datum surface (see Section 2.6) and the number of teeth (see Section 2.4).

In general a distinction is made between the normal module m_n (in a normal section of the gear tooth system, see Section 7.3.2), the transverse module m_t (in a transverse section, see Section 7.3.1) and the axial module m_x (in an axial section, see Section 7.3.3).

2.4 Number of teeth z

The number of teeth z of a gear is the number of teeth present on the full circumference of the gear or feasible for a chosen pitch. For sign of the number of teeth see Section 2.9.

2.5 Gear axis

The axis of a gear (gear axis) is the axis of its bore or the common axis of its journals.

2.6 Datum surface, reference surface

The datum surface of a gear tooth system is a notional surface to which the geometrical parameters of the gear teeth are referred. Apart from the limiting cases of Sections 5.1.2 and 5.1.4 and with the exception of non-round gears, it is a surface of rotation about the gear axis.

In the case of rolling type gears (see Section 4.3) the datum surfaces are designated as reference surfaces (reference cylinder or reference cone). Their generators are straight lines; see Fig. 5.

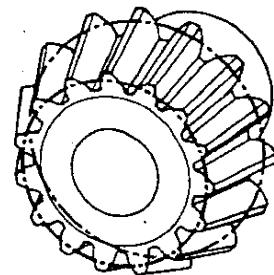
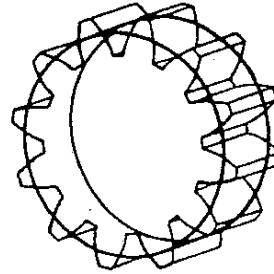


Figure 5. Reference cylinder of a cylindrical gear, reference cone of a bevel gear

2.7 Plane tooth system

A tooth system with a plane datum surface (see e. g. rack, Section 5.1.2, or plane bevel gear, Section 5.1.4) is designated as a plane tooth system.

2.8 Axial boundary

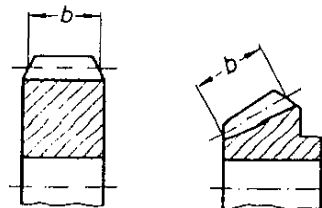
2.8.1 End surfaces of the gear tooth system

The end surfaces of a gear tooth system are its boundary surfaces normally at right angles to the envelope lines of its datum surface.

Note: Cylindrical gears generally have plane end surfaces which cut the gear axes perpendicularly (in special cases the end surfaces are cones about the gear axis). Bevel gears usually have conical end surfaces; in the case of crown wheels the end surfaces are normally cylinders about the gear axis.

2.8.2 Facewidth b

The facewidth b is the distance between the two end surfaces on the datum surface of a gear tooth system, see Fig. 6.

Figure 6. Facewidth b

2.9 Position of the gear tooth system relative to the gear body

2.9.1 External gear teeth, external gear

With external gear teeth the teeth project outwards from the gear body (away from the gear axis).

A gear with external teeth is termed an external gear or externally-toothed gear.

In calculations the number of teeth of an external gear is entered in the equations as a positive quantity.

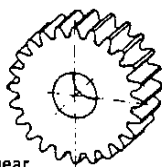


Figure 7. External gear

2.9.2 Internal gear teeth, internal gear

With internal gear teeth the teeth project inwards from the gear body (towards the gear axis).

A gear with internal teeth is termed an internal gear or internally-toothed gear.

In calculations the number of teeth of an internal gear is entered in the equations as a negative quantity. This means that all quantities derived from it – e. g. all diameters and radii, angular pitch, gear ratio and centre – also take a negative sign.

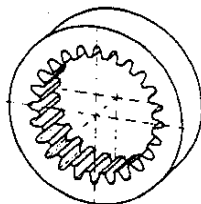


Figure 8. Internal gear

3 General terms concerning gear pairs

3.1 Gear pair

A gear pair is a simple mechanism consisting of two gears the axes of which are in a defined position relative to each other so that the first gear transmits its rotation to the second gear by way of the teeth which come successively into engagement (see Section 3.8).

Gears with circular (axially symmetrical) datum surfaces which are concentric with the gear axis effect a uniform transmission of rotary motion; gears with non-round or eccentric datum surfaces effect a transmission of the rotary motion which varies periodically.

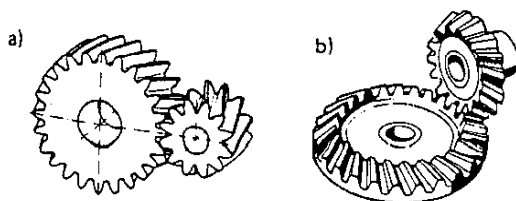


Figure 9. Gear pair

a) Cylindrical gear pair

b) Bevel gear pair

3.1.1 Gear and mating gear

One of the two gears of a gear pair is arbitrarily designated as the gear and the gear meshed with it as the mating gear.

3.1.2 Pinion (small gear) and wheel (large gear)

The smaller of the two gears constituting a gear pair is designated as the pinion or small gear whilst the larger is termed the wheel or large gear. The smaller gear is denoted by the subscript 1 and the larger by the subscript 2.

With certain types of gear pairs the small gear and the large gear are given special names (e. g. worm and worm-wheel, see Sections 5.2.1 and 5.3).

3.1.3 Driving and driven gear

The gear of a gear pair which drives the other is termed the driving gear. The gear which is driven by the other gear is the driven gear. The driving gear is denoted by the subscript a and the driven gear by the subscript b.

3.1.4 Multiple gear-pairing, gear train

A gear train is a combination of two or more gear pairs which are actively related to one another.

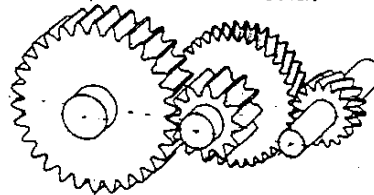


Figure 10. Gear train

3.2 Gear transmission

A gear transmission is an assembly consisting of one or more gear pairs and the housing or frame (usually enclosing the gear pairs) which mounts the bearings for the stationary gear axes. In a gear transmission the magnitude and/or direction of the rotary motion and torque can be transformed in one or more stages.

3.2.1 Single-stage gear transmission

A single-stage gear transmission contains a gear pair consisting of a gear and mating gear mounted in a housing (frame) together with the bearings for both gear axes.

3.2.2 Multi-stage gear transmission

A multi-stage (two-stage, three-stage etc.) gear transmission contains several (two, three etc.) gear pairs (gear trains) arranged successively in the direction in which the rotary motion is transmitted, and mounted in a common housing.

3.2.3 Fixed-axis gear transmission

A fixed-axis gear transmission is a single-stage or multi-stage gear transmission in which all the rotatably mounted gear axes occupy non-varying positions.

3.2.4 Epicyclic or planetary gear transmission (gear train)

An epicyclic or planetary gear transmission is one having at least three gears arranged in sequence in the effective direction, such that the shafts of two of the gears are arranged coaxially whilst the third or intermediate gear (epicyclic or planet gear) is mounted on a rotating arm

(planet carrier) coaxial with the gear shaft and rotates with the arm.

An epicyclic or planetary gear train thus consists generally of the externally-toothed central gear (sun gear, central pinion), one (or more parallel-mounted) externally-toothed planet gear(s) and the internal gear (usually non-rotatable) which is coaxial with the sun gear.

In special cases an external gear may be used instead of the internal gear. With this arrangement the rotating shaft of the arm carries two externally-toothed intermediate gears fixed together. The epicyclic gear train thus consists of two external gear pairs arranged in succession in the effective direction, the two external gears which are not fixed together being coaxial.

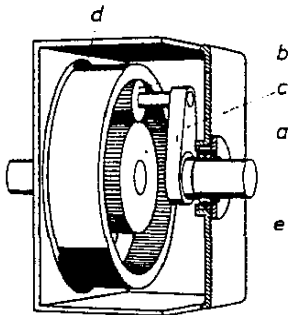


Figure 11. Epicyclic or planetary gear transmission

- a Sun gear
- b Planet gear
- c Rotating arm
- d Internal gear
- e Housing

3.3 External gear pair

An external gear pair is a gear pair in which both gears are externally-toothed (external gears).

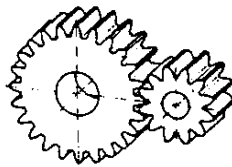


Figure 12. External gear pair

3.4 Internal gear pair

An internal gear pair is a gear pair in which one of the two gears is internally-toothed (an internal gear).

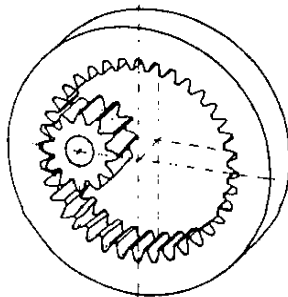


Figure 13. Internal gear pair

3.5 Gear axes of a gear pair

The forms assumed by the gears of a gear pair and the shapes of the datum surfaces of their tooth systems depend on the mutual position of the gear axes. The two gear axes may lie in the same plane, in which case they may be parallel or intersect each other, or they may not lie in the same plane and cross each other.

The mutual position of the gear axes determines the following parameters:

3.5.1 Axial plane

In the case of a gear pair with parallel or intersecting axes, the latter determine the axial plane.

3.5.2 Crossing line, crossing points

In the case of a gear pair with non-parallel non-intersecting axes the common perpendicular to both axes is the crossing line. The point of intersection of the crossing line with one of the axes is a crossing point.

3.5.3 Mid-planes

In the case of a gear pair with non-parallel non-intersecting axes one of the axes and the crossing line determine a mid-plane. A gear pair with non-parallel non-intersecting axes thus has two mid-planes which intersect in the crossing line.

3.5.4 Centre distance a (offset)

The centre distance a of a gear pair with parallel or non-parallel non-intersecting axes is the shortest distance between the two axes. In the case of a gear pair with non-parallel non-intersecting axes it lies in the crossing line.

In the case of a hypoid gear pair (see Section 5.2.2) the centre distance is also termed the offset.

Note: In calculations involving an internal gear pair with parallel axes the centre distance is a negative quantity, see Section 2.9.2.

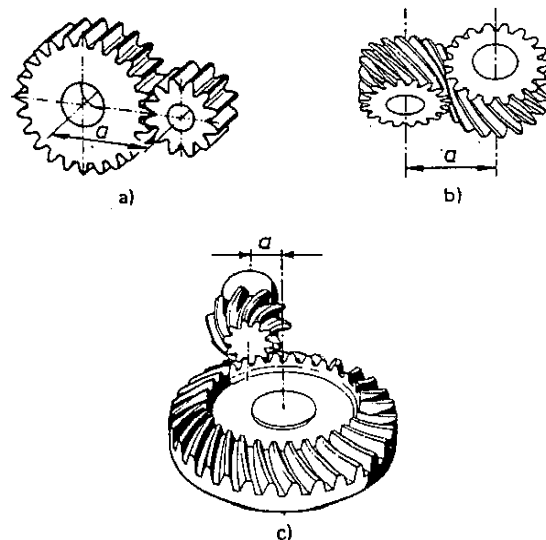


Figure 14. Centre distance

- a) With parallel gear axes
- b) and c) with non-parallel non-intersecting axes

3.5.5 Shaft angle Σ

The shaft angle Σ of a gear pair with intersecting axes is the angle through which one of the two axes must be swung beyond the instantaneous axis to bring the axes into coincidence.

Note: In the case of externally-toothed bevel gears the shaft angle is positive. Internally-toothed bevel gears do not arise in practice. In calculations involving them the shaft angle would be negative.

The shaft angle Σ of a gear pair with non-parallel non-intersecting axes is the smaller of the two angles between the mid-planes. It is positive if, when viewing from gear axis 1 to gear axis 2, the mid-plane containing the axis of gear 2 must be swung in the clockwise direction about the crossing line so that two gear axes assume a parallel attitude and the gears have opposite directions of rotation. If, on the other hand, it has to be swung in the counter-clockwise direction then the shaft angle is negative; see Figs. 16, 17 and 24.

3.6 Gear ratio u

The gear ratio u of a gear pair is the ratio of the number of teeth z_2 of the larger gear to the number of teeth z_1 of the smaller gear:

$$u = \frac{z_2}{z_1}, \quad (1)$$

where $|z_2| \geq z_1$ and hence $|u| \geq 1$.

Note: The gear ratio of an external gear pair is positive, whilst that of an internal gear pair is negative, see Section 2.9.

3.7 Transmission ratio i

The transmission ratio i of a gear pair or of a multiple gear-pairing is the ratio of the angular velocity ω_a (or of the speed n_a) of the first driving gear to that of the last driven gear ω_b (or n_b):

$$i = \frac{\omega_a}{\omega_b} = \frac{n_a}{n_b} \quad (2)$$

Where necessary the transmission ratio is designated as positive if the angular velocities have the same direction of rotation, and as negative if they have opposite directions of rotation.

Note: A rolling type gear pair (see Section 4.3) consisting of two external gears reverses the direction of rotation; one of the gears has negative rotation and the transmission ratio is negative. In the case of a rolling type gear pair consisting of an external gear and an internal gear, both gears have the same direction of rotation and their transmission ratio is positive.

In the case of non-uniform angular velocities (e. g. in the case of non-round or eccentric gears or departures of the tooth systems from their nominal dimensions, or in the case of special flank forms) a distinction should be made if necessary between the instantaneous transmission ratio

$$i = \frac{\omega_a}{\omega_b} \text{ and the mean transmission ratio } i_m = \frac{n_a}{n_b}.$$

3.7.1 True-angle ratio

If the angular velocities of the driving gear and driven gear are uniform (i. e. if the instantaneous transmission

ratio and the mean transmission ratio are in continuous agreement) the ratio is said to be uniform or true-angle.

3.7.2 True-torque ratio

If the torques of the driving gear and driven gear are uniform the transmission ratio is said to be true-torque.

3.7.3 Speed-reducing ratio

With $|i_m| > 1$ the transmission ratio is said to be speed-reducing.

3.7.4 Speed-increasing ratio

With $|i_m| < 1$ the transmission ratio is said to be speed-increasing.

3.8 Engagement

The cooperation of a gear with its mating gear is termed engagement.

3.8.1 Point of contact

When the gears are in a certain position the profiles of the flank and mating flank (see Section 7.5.1) touch each other at the point of contact. As the gears rotate the point of contact moves along the profile.

3.8.2 Contact line

The contact line is the totality of those points on a tooth flank at which there is simultaneous contact each time with the mating flank. As the gears rotate the contact line moves over the active range of the tooth flanks, see Section 7.6.3.

3.8.3 Plane of action

The plane of action is the geometrical locus of all points of contact of the tooth flanks which are imagined as being unbounded.

A gear pair has two planes of action, one for the right flanks and one for the left flanks, see Section 7.5.2.

3.8.4 Zone of action

The zone of action is the geometrical locus of all points of contact belonging to the active range of the tooth flanks (see Section 7.6.3). It is that part of the plane of contact within which engagement takes place while the gear transmission is in operation.

3.8.5 Path of contact

In the case of a rolling type gear transmission (see Section 4.3) a path of contact is the line of intersection of the plane of contact with a plane perpendicular to the instantaneous axis (in the case of parallel axes) or with a sphere centred on the point of intersection of axes (in the case of intersecting axes).

3.8.6 Length of path of contact

The length of the path of contact is that part of the path of contact which belongs to the active range of the tooth flanks (see Section 7.6.3).

3.8.7 Meshing interference

Meshing interference is an unwanted contact disturbing the theoretical engagement between a tooth and the mating tooth.

3.9 Types of tooth system

3.9.1 Individual tooth system

In an individual tooth system the tooth system of a given gear determines the tooth system of its mating gear or its mating gears.

Note: A cylindrical worm (see Section 5.2.1) or an enveloping worm (see Section 5.3) or a cylindrical lantern gear (see Section 7.8.3) determine, for example, the tooth systems of their mating gears.

3.9.2 Paired tooth system; X-zero tooth system

In a paired tooth system the teeth of gear and mating gear are determined by an assumed plane tooth system and its mating plane tooth system, which match each other like a punch and die, or by assumed paths of contact.

If the two matching plane tooth systems are identical, or if the paths of contact are symmetrical to the crossing line (centre distance line) of the gear pair, then gears differing in number of teeth can be produced by a single cutting tool and mated with each other as desired. Such tooth systems are termed X-zero tooth systems.

4 Kinematic terms

4.1 Instantaneous axis

The instantaneous axis is that line of a gear pair which is to be regarded as the axis for the instantaneous motion of one of the gears relative to the other gear which is imagined as being stationary.

4.2 Functional surfaces

The functional surfaces of a gear pair are imaginary surfaces (usually surfaces of rotation) about the gear axes which, as non-toothed surfaces, perform the same relative motions as the gears. The functional surfaces of the gear pairs are the pitch surfaces (see Section 4.3.4) in the case of rolling type gear transmissions according to Section 5.1 and the helical pitch surfaces (see Section 4.4.3) in the case of helical rolling type gear transmissions according to Sections 5.2.2 to 5.2.4.

4.3 Rolling type gear transmissions

Rolling type gear transmissions are transmissions in the functional surfaces of which a pure rolling action (without sliding) occurs.

Note: Rolling type gear transmissions comprise those gear pairs whose axes lie in a single plane (parallel or intersecting).

4.3.1 Rolling axis

The instantaneous axis of a rolling type gear transmission is termed the rolling axis, see Fig. 15. The relative instantaneous rotation of one of the gears about the other gear takes place around this axis (without longitudinal displacement of the axis). The rolling axis lies in the axial plane of the gear pair.

4.3.2 Plane gear transmissions, spherical gear transmissions

Rolling gear transmissions with parallel gear axes are termed plane gear transmissions. In this case the parts of the tooth systems which come into mesh with one

another lie in planes perpendicular to the gear axes. The rolling axis lies parallel with the gear axes.

Rolling type gear transmissions with intersecting axes are termed spherical gear transmissions. In this case the parts of the tooth systems which come into mesh with one another lie on spherical surfaces centred on the point of intersection of the axes. The rolling axis passes through the point of intersection of the axes. If the point of intersection of the axes moves to infinity the spherical gear transmission becomes a plane gear transmission.

4.3.3 Pitch point C

Each point of the rolling axis in the area of the usable flanks is a pitch point C. The ratio of the distances of a pitch point to the axes of pinion and wheel is equal to the reciprocal of the gear ratio u . The position of the rolling axis in the axial plane is thus determined by the mutual position of the gear axes (centre distance a or shaft angle Σ) and by the ratio u .

4.3.4 Pitch surface; pitch cylinder, pitch cone

The rolling axis in its relative rotation about the gear axes describes the pitch surfaces of the gear and its mating gear. The two pitch surfaces (functional surfaces) touch each other in the prevailing rolling axis.

The pitch surfaces of a gear pair with parallel axes are cylinders, namely the pitch cylinders.

The pitch surfaces of a gear pair with intersecting axes are cones, the apices of which lie at the point of intersection of the axes, namely the pitch cone.

When paired with several mating gears a gear may have several pitch surfaces.

During its generation (mating with the cutter) a gear may have a different pitch surface from that which exists in the gear transmission (paired with the mating gear). A distinction is therefore made between the generating pitch surface and the working pitch surface.

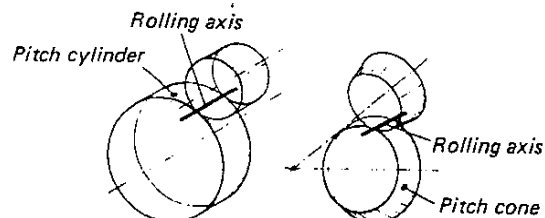


Figure 15. Rolling axis, pitch surfaces

4.4 Helical rolling type gear transmissions

Helical rolling type gear transmissions contain a gear pair with non-parallel non-intersecting axes the gears of which engage with a screw-like motion and additionally possess either one or two rolling capabilities resulting from the special shape of their gear bodies and tooth flanks.

A single rolling possibility is possessed by a gear pair consisting of a cylindrical worm and a double enveloping wormwheel according to Section 5.2.1: if the wormwheel is rotatably mounted in fixed bearings and the cylindrical worm is moved axially (without rotating) the two gears roll relatively to one another without any relative tangential sliding of the teeth taking place.

Two rolling options are possessed by the crossed helical and hypoid gear pairs according to Sections 5.2.3 and 5.2.4: the ideal plane tooth systems of the two gears can

be imagined to lie in the common plane through the helix point (see Section 4.4.2) perpendicular to the crossing line, and each of the two gears possesses one rolling possibility on the related plane tooth system.

4.4.1 Helix axis

The instantaneous axis of a helical rolling type gear transmission is termed the helix axis. About this axis there takes place instantaneously a screw-like motion of one of the gears about the other gear which is imagined to be stationary, i. e. a rotary motion with relative angular velocity ω_{rel} (see Section 4.4.4.1) and simultaneously a translatory motion along the helix axis with the sliding velocity v_{gs} (see Section 4.4.4.2). The helix axis intersects the crossing line at right angles.

Parameters on the helix axis are denoted by the subscript s.

4.4.2 Helix point S

The point of intersection of the helix axis with the crossing line is the helix point S. This divides the centre distance a into the two sections r_{s1} and r_{s2} depending on the gear ratio u and the shaft angle Σ according to the equation

$$r_{s1} : r_{s2} = (1 + u \cdot \cos \Sigma) : (u^2 + u \cdot \cos \Sigma) \quad (3)$$

whence it follows that

$$r_{s1} = a \cdot (1 + u \cdot \cos \Sigma) : (1 + 2 \cdot u \cdot \cos \Sigma + u^2) \quad (4)$$

$$r_{s2} = a \cdot (u^2 + u \cdot \cos \Sigma) : (1 + 2 \cdot u \cdot \cos \Sigma + u^2) \quad (5)$$

4.4.3 Helical pitch surfaces

The helix axis in its relative rotation about the two gear axes describes the helical pitch surfaces of the gear and its mating gear. These functional surfaces are single-shell

hyperboloids with radii r_{s1} and r_{s2} and they contact each other in the relevant helix axis.

4.4.4 Parameters at the helix point S

4.4.4.1 Relative angular velocity ω_{rel}

The relative angular velocity ω_{rel} with which a gear of a helical rolling type gear transmission instantaneously rotates about the mating gear imagined as being stationary derives from the vector parallelogram of the angular velocities of the two gears, see Fig. 16.

Note: Fig. 16 shows in plan the projections of the two gear axes 1-1 and 2-2 on a plane parallel to them. From the projection of the crossing point O_1 (on the axis of gear 1) the vector of the angular velocity is $\vec{\omega}_1 = \vec{O_1 W_1}$. From the projection of the crossing point O_2 (on the axis of gear 2; its projection coincides with O_1) the vector of the angular velocity $\vec{\omega}_2 = \vec{O_2 W_2}$ and the vector of the angular velocity $-\vec{\omega}_2 = \vec{O_1 W_2}$ are plotted. The angle $W_1 O_2 W_2$ is equal to the shaft angle Σ which in this case has a positive sign.

In the vector parallelogram $O_1 W_1 W_{rel} W_2$ the diagonal $\vec{O_1 W_{rel}}$ has the same direction as the helix axis. The angle enclosed by the vectors $\vec{\omega}_1 = \vec{O_1 W_1}$ and $\vec{\omega}_{rel} = \vec{O_1 W_{rel}}$ is the crossing angle β_{s1} between the axis of gear 1 and the helix axis. The angle enclosed by the vectors $-\vec{\omega}_2 = \vec{O_1 W_2}$ and $\vec{\omega}_{rel} = \vec{O_1 W_{rel}}$ is the crossing angle β_{s2} between the axis of gear 2 and the helix axis. The following applies: $\beta_{s1} + \beta_{s2} = \Sigma$ (6)

$$\sin \beta_{s2} = u \cdot \sin \beta_{s1} = \frac{u \cdot \sin \Sigma}{\sqrt{1 + 2 u \cos \Sigma + u^2}} \quad (7)$$

These helix angles are positive if right-handed and negative if left-handed. If both of the helix angles are positive then the shaft angle also is positive; if both are negative then the shaft angle also is negative.

The diagonal $\vec{O_1 W_{rel}}$ represents the vector of the relative angular velocity ω_{rel} in the scale of the vectors of the angular velocity ω_1 and ω_2 .

The following applies:

$$\omega_{rel} = \omega_1 \cdot \frac{\sin |\Sigma|}{\sin |\beta_{s2}|} = \omega_2 \cdot \frac{\sin |\Sigma|}{\sin |\beta_{s1}|} \quad (8)$$

The relative angular velocity ω_{rel} of gear 1 in relation to gear 2 is denoted by ω_{12} whilst that of gear 2 in relation to gear 1 is denoted by ω_{21} . Their absolute amounts and their vectors are equal, but oppositely directed:

$$\vec{\omega}_{21} = -\vec{\omega}_{12} \quad (9)$$

4.4.4.2 Velocity of sliding v_{gs}

The velocity of sliding v_{gs} along the helix axis has the following magnitude

$$v_{gs} = a \cdot \omega_1 \cdot \sin |\beta_{s1}| = a \cdot \omega_2 \cdot \sin |\beta_{s2}| \quad (10)$$

The relative velocity of sliding v_{gs} of gear 1 versus gear 2 is denoted by v_{gs12} whilst that of gear 2 versus gear 1 is denoted by v_{gs21} . The two are of equal magnitude, but oppositely directed, see Fig. 16.

$$\vec{v}_{gs12} = -\vec{v}_{gs21} \quad (11)$$

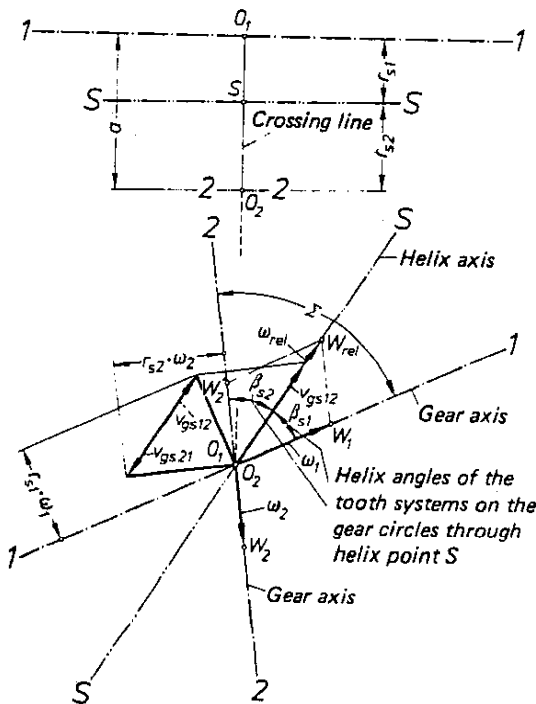


Figure 16. Kinematic parameters at the helix point S

4.4.4.3 Reduced lead (parameter)

$p_{zs\ red}$ of the helical motion
The reduced lead (parameter) $p_{zs\ red}$ of the helical motion is

$$p_{zs\ red} = \frac{v_{gs}}{\omega_{rel}} = a \cdot \frac{\sin|\beta_{s1}| \cdot \sin|\beta_{s2}|}{\sin|\Sigma|} \quad (12)$$

4.4.5 Working point W

In the case of cylindrical worm gear sets (see Section 5.2.1 and DIN 3975) with large gear ratio u the distance r_{s1} according to equation (4) is very small whilst the distance r_{s2} according to equation (5) is very large. The helix point S is therefore very close to the wormwheel axis and within the gear body. The angles β_{s1} and β_{s2} are thus meaningless for the tooth systems of the gear pair. This being so, the helix point S can be superseded by a working point W on the crossing line, and the helix axis can be superseded by the instantaneous axis parallel to the helix axis and passing through W.

Parameters at the working point are denoted by the subscript w.

The position of the working point can be chosen freely. The dimensions of the worm and wormwheel are then determined accordingly.

On the other hand, if the dimensions of the worm $p_{z1\ red}$ and β_{w1} are known or specified they can be used for calculating the position of the working point on the crossing line for a given position of the gear axes (a and Σ) and a specified gear ratio.

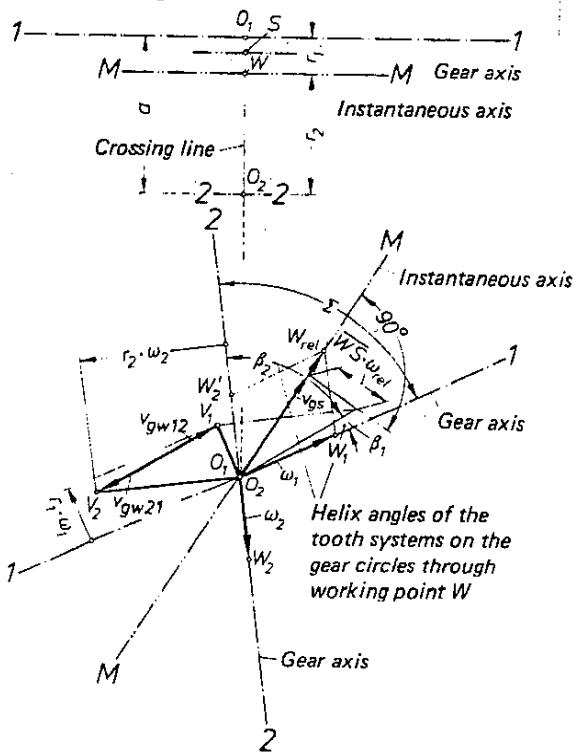


Figure 17. Kinematic parameters at the working point W

Distance WO_2 :

$$WO_2 = r_2 = p_{z1\ red} \cdot \frac{z_2}{z_1} \cdot \frac{\sin|\beta_{w1}|}{\cos(\Sigma - |\beta_{w1}|)} \quad (13)$$

Distance WO_1 :

$$WO_1 = r_{w1} = a - r_2. \quad (14)$$

where

r_2 Pitch circle radius = reference circle radius of the wormwheel (see DIN 3975, October 1976 edition, Section 4.4.1),

$p_{z1\ red}$ reduced lead of worm (see DIN 3975, October 1976 edition, Section 3.4.2),

β_{w1} helix angle of worm at radius $r_{w1} = r_{m1} + x \cdot m$ (see DIN 3975, October 1976 edition, Sections 3.5.1 and 4.4.3).

If the worm functions as a rack in a rolling type gear transmission (see DIN 3975, October 1976 edition, Section 2.7) the rolling axis (see DIN 3975, Section 2.6.3) passes parallel to the wormwheel axis through the working point (pitch point) W.

4.4.6 Parameters at the working point W

4.4.6.1 Relative angular velocity ω_{rel}

The relative angular velocity ω_{rel} is found in the same way as in Section 4.4.4.1.

4.4.6.2 Velocity of sliding v_{gw}

The magnitude and direction of the velocity of sliding v_{gw} depend on the position of the working point W on the crossing line.

The velocity of sliding v_{gw} has the following magnitude

$$v_{gw} = v_{w1} \cdot \sin|\beta_{w1}| + v_{w2} \cdot \sin|\beta_{w2}| = v_{w1} \cdot \frac{\sin|\Sigma|}{\cos\beta_{w2}} = v_{w2} \cdot \frac{\sin|\Sigma|}{\cos\beta_{w1}} \quad (15)$$

where

v_{w1} and v_{w2} are the circumferential velocities of the two gears in the working point W,

β_{w1} and β_{w2} are the helix angles of the gear tooth systems at the radii r_{w1} and r_{w2} .

The relative velocity of sliding v_{gw} of gear 1 versus gear 2 is denoted by v_{gw12} whilst that of gear 2 versus gear 1 is denoted by v_{gw21} . The two are of equal magnitude but oppositely directed:

$$v_{gw12} = -v_{gw21} \quad (16)$$

Note: The direction of the velocity of sliding does not correspond with the direction of the instantaneous axis through W. The velocity of sliding v_{gw} has a component of magnitude v_{gs} in the direction of the instantaneous axis and a component of magnitude $WS \cdot \omega_{rel}$ at right angles to the instantaneous axis and to the crossing line.

The following expression applies:

$$v_{gw} = \sqrt{v_{gs}^2 + (WS \cdot \omega_{rel})^2} \quad (17)$$

If W coincides with S then $v_{gw} = v_{gs}$. The smallest possible velocity of sliding in a helical rolling type gear transmission is assigned to the helix axis.

In Fig. 17 the projections of the vectors of the circumferential velocities of gears 1 and 2 at point W $\vec{v}_{w1} = r_1 \cdot \omega_1 = \vec{O_1V_1}$ and $\vec{v}_{w2} = r_2 \cdot \omega_2 = \vec{O_1V_2}$ are plotted from the projection of the crossing points O_1 and O_2 . The vector $\vec{V_1V_2}$ between the end points of these vectors represents the direction and magnitude of the

relative velocity of sliding \vec{v}_{gW} to the scale of the circumferential velocities \vec{v}_{w1} and \vec{v}_{w2} .

The crossing angle β_{w1} of vector \vec{v}_{gW} with the axis of gear 1 and the crossing angle β_{w2} of vector \vec{v}_{gW} with the axis of gear 2 are the helix angles of the two gear tooth systems on their coaxial cylinders with radii $O_1 W$ or $O_2 W$.

4.5 Pure helical type gear transmissions

In pure helical type gear transmissions the two gears engage each other with a screw-like motion only and possess no relative rolling capability.

Pure helical type gear transmissions — like helical rolling type gear transmissions — have non-parallel non-intersecting axes. The double enveloping worm gear sets according to Section 5.3 belong to this type of transmission.

5 Types of gears and gear pairs

5.1 Gears and gear pairs for rolling type gear transmissions

5.1.1 Cylindrical gear; cylindrical gear pair

A cylindrical gear is a gear of which the datum surface (reference surface) is a circular cylinder which has diameter d (radius r) and is termed the reference cylinder.

The pairing of two cylindrical gears yields a cylindrical gear pair. Their axes are parallel and they have the centre distance a . The reference cylinders of the gear tooth systems may coincide with the pitch surfaces of the gear pair.

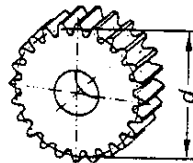


Figure 18. Cylindrical gear with reference diameter d

5.1.2 Rack; rack and pinion pair

A rack is the limiting case of an externally toothed cylindrical gear of infinitely large diameter. Its datum surface is a plane and its tooth system is a plane tooth system. Each plane parallel to the datum plane is a reference plane.

The mating of a rack with a spur gear gives a rack and pinion pair. In this case the reference cylinder of the cylindrical gear is at the same time the pitch cylinder; the reference plane of the rack contacting the reference cylindrical gear is the pitch plane.

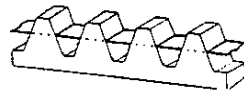


Figure 19. Rack with reference plane

5.1.3 Bevel gear; bevel gear pair

A bevel gear is a gear of which the datum surface (reference surface) is a circular cone which has the half-angle δ and is termed the pitch cone.

The mating of two bevel gears yields a bevel gear pair. Their axes intersect at the axis intersection point and enclose the shaft angle Σ . The reference cones of the gear tooth systems generally coincide with the pitch cones of the gear pair.

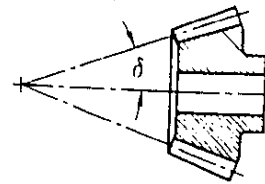


Figure 20. Bevel gear with reference cone angle δ

5.1.4 Plane bevel gear; plane bevel gear pair

A plane bevel gear is the limiting case of an externally toothed bevel gear the half cone angle of which $\delta_2 = 90^\circ$. Its datum surface is a plane at right angles to the gear axis. Its tooth system is a plane tooth system disposed on an end face of the gear.

The mating of a plane bevel gear with a bevel gear yields a plane bevel gear pair. The shaft angle is generally $\Sigma = 90^\circ + \delta_1$.

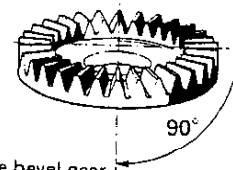


Figure 21. Plane bevel gear

5.1.5 Contrate gear pair; crown wheel

A contrate gear pair consists of the mating of a cylindrical gear with a plane gear at a shaft angle $\Sigma = 90^\circ$. The pitch surfaces are cones. The cylindrical tooth system and the gear ratio determine the plane tooth system. In light precision engineering in particular the plane gear is also termed a crown wheel.

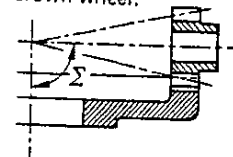


Figure 22. Contrate gear pair

5.2 Gears and gear pairs for helical rolling type gear transmissions

5.2.1 Cylindrical worm gear set (worm gear set); cylindrical worm (worm), wormwheel

A cylindrical worm gear set (worm gear set) consists of a gear with cylindrical datum surface, namely the cylindrical worm (worm), and the corresponding enveloping mating gear, namely the wormwheel (see DIN 3975. The shaft angle is usually $\Sigma = 90^\circ$. Other shaft angles are feasible. The tooth systems of worm and wormwheel make line contact in a zone of action.

A worm gear set with shaft axes crossing at right angles can also function as a rolling type gear transmission (like a rack and pinion, see DIN 3975, October 1976 edition, Section 2.7).

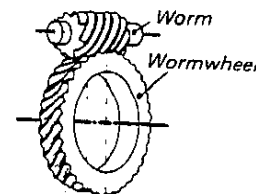


Figure 23. Cylindrical worm gear set (worm gear set)

5.2.2 Hyperboloid gear pair

A hyperboloid gear pair has two gears the tooth systems of which lie in the hyperboloid functional surfaces. They may be designed as concave gears (hyperboloid cylindrical gears with tooth systems about the helix point) or as hyperboloid bevel gears the tooth systems of which lie outside the helix point.

In general these tooth systems are replaced by tooth systems approximating to them: crossed helical gear pair (cylindrical helical gear pair) according to Section 5.2.3 or hypoid gear pair (crossed helical bevel gear pair) according to Section 5.2.4.

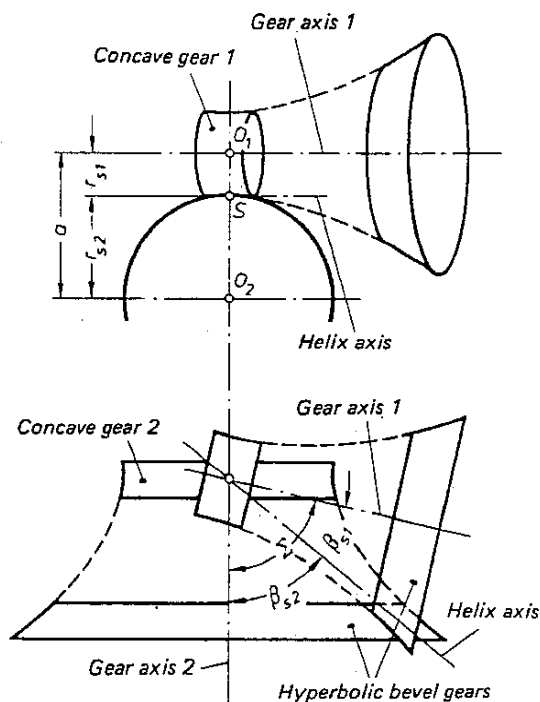


Figure 24. Hyperboloid gears

5.2.3 Crossed helical gear pair (cylindrical crossed helical gear pair); crossed helical gears (cylindrical crossed helical gears), crossed helical worm gear

With a sufficiently large centre distance a and small gear ratio u (normally $u < 4$) the radii r_{s1} and r_{s2} of the helical pitch surfaces are sufficiently large to enable the hyperboloids to be approximated by cylindrical surfaces in a narrow region about the helix point. In this case, therefore, suitably toothed cylindrical gears can be paired with one another instead of the hyperboloid gears. They are then termed crossed helical gears and yield a crossed helical gear pair. Their tooth systems have only point contact; under operating conditions the contact point develops into a contact surface.

In light precision engineering crossed helical gear pairs are also used with relatively large gear ratio ($u > 4$) and with very small number of teeth z_1 . The smaller crossed helical gear then has a small diameter with comparatively large facewidth. In its geometry it resembles a cylindrical worm and is therefore also termed a crossed helical worm gear.

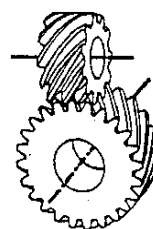


Figure 25. Crossed helical gear pair

5.2.4 Hypoid gear pair (helical bevel gear pair); hypoid gears (helical bevel gears)

Assuming a small centre distance a (usually termed offset) it is possible to use for meshing purposes those parts of the helical pitch surfaces which are sufficiently distant from the helix point to be capable of being approximated by conical surfaces. The pitch cones of the bevel gears differ from the corresponding parts of the helical pitch surfaces; they are so chosen that the tooth engagement extends over nearly the entire facewidth. These bevel gears are termed hypoid gears (helical bevel gears); their mating yields a hypoid gear pair (helical bevel gear pair). The shaft angle is usually $\Sigma = 90^\circ$.

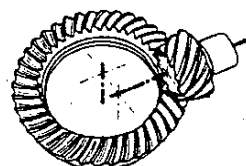


Figure 26. Hypoid gear pair (helical bevel gear pair)

5.3 Double enveloping worm gear set; double enveloping worm, double enveloping wormwheel

A double enveloping worm gear set is a gear pair with gear axes crossing at right angles and consisting of two toothed globoids, the enveloping worm and the enveloping wormwheel. The datum surfaces of the two gear tooth systems are globoids which are so related that the trace of the one is the generating circle of the other. Tooth engagement takes place only in the central plane of the enveloping wormwheel.

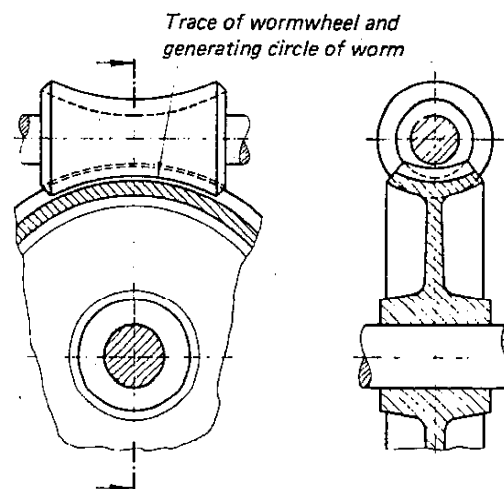


Figure 27. Double enveloping worm gear set

6 Crest and root surfaces

6.1 Crest

The crest of a tooth is the outermost boundary surface of the tooth coaxial with the datum surface of the tooth system.

6.2 Tip surface

The tip surface of a gear is the surface which is coaxial with the datum surface of the tooth system and contains all the crests of the teeth.

6.3 Bottom of tooth space

The bottom of the tooth space is the inner boundary surface of a tooth space which is coaxial with the datum surface of the tooth system.

6.4 Root surface

The root surface of a gear is the surface coaxial with the datum surface of the tooth system which contains all the surfaces at the bottom of the tooth spaces.

7 Tooth flanks and tooth profiles

7.1 Tooth flanks

The tooth flanks are those parts of the surface of a tooth which extend between the tip surface and the root surface.

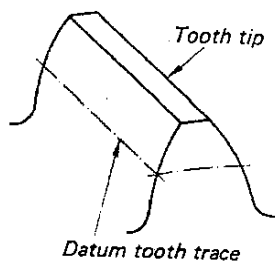


Figure 28. Tooth flank

7.2 Datum tooth trace, tooth trace, reference tooth trace

The datum tooth trace (or tooth trace, reference tooth trace) is the intersection of a tooth flank with the datum surface (or with a surface coaxial with the datum surface or with the reference surface).

7.3 Gear tooth profile, tooth profile, flank profile

The gear tooth profile (or tooth profile or flank profile) is the intersection of the tooth system (or of a tooth or a tooth flank) with an assumed, non-coaxial surface of intersection.

7.3.1 Transverse profile; transverse section

The transverse profile is the profile of the tooth system (or of a tooth or a tooth flank) in the transverse section.

A transverse section is a surface at right angles to the generator of the datum surface. Parameters in a transverse section are denoted by the subscript *t*.

Note: In the case of a cylindrical gear (crossed helical gear, worm) the transverse section is at right angles to the gear axis.

7.3.2 Normal profile; normal section

The normal profile is the profile of the tooth system (or of a tooth or a tooth flank) in the normal section.

A normal section is a surface at right angles to the tooth traces. It generally possesses three-dimensional curvature. Parameters in a normal section are denoted by the subscript *n*.

7.3.3 Axial profile; axial section

The axial profile is the profile of the tooth system (or of a tooth or a tooth flank) in the axial section.

An axial section is a plane containing the gear axis. Parameters in an axial section are denoted by the subscript *x*.

7.4 Standard basic rack tooth profile

The standard basic rack tooth profile is a profile of a plane tooth system defined by agreement which, in conjunction with other specification factors, determines the tooth systems of the associated gears.

7.5 Types of tooth flank

7.5.1 Mating flanks

Mating flanks of the tooth systems of a gear pair are those flanks of the two gears which come into engagement with each other or may come into engagement.

7.5.2 Right flank, left flank

The right flank (or left flank) is that flank which an observer viewing from an agreed direction sees on the right (or left) side of an upward-directed tooth.

Note: This definition applies equally to external and internal gears.

In the case of cylindrical or bevel gear pairs — assuming a common viewing direction, e. g. towards the point of intersection of the axes — right flanks cooperate with right flanks and left flanks with left flanks.

7.5.3 Corresponding flanks

Corresponding flanks comprise all the right flanks of a gear on the one hand, and all the left flanks on the other.

7.5.4 Opposite flanks

Opposite flanks of a gear comprise one or more right flanks as opposed to one or more left flanks, or vice versa.

7.5.5 Working flanks

Working flanks are those flanks by means of which the motion of one of the gears is transmitted to the mating gear or received from the latter.

7.5.6 Non-working flanks

Non-working flanks are the flanks of a gear which are opposite the working flanks.

7.6 Parts of flanks

7.6.1 Addendum flank, dedendum flank

The addendum flank (or dedendum flank) is that part of a flank which lies between the datum surface and the tip surface (or root surface).

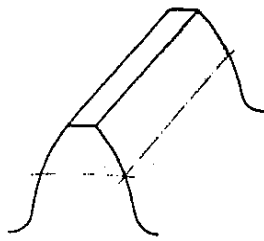


Figure 29. Addendum flank

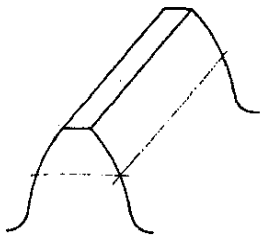


Figure 30. Dedendum flank

7.6.2 Usable flank; usable tip circle, usable root circle

The usable flank is that part of a tooth flank which may be used for engagement with a mating flank.

The usable flank is bounded in the radial direction by the usable tip circle with diameter d_{Na} and the usable root circle with diameter d_{Nr} .

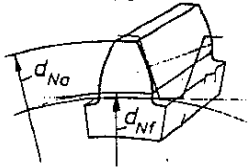


Figure 31. Usable flank

7.6.3 Active flank

The active flank is that part of a tooth flank which comes into engagement with the mating flanks.

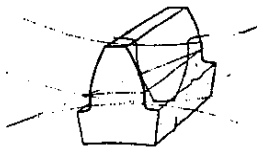


Figure 32. Active flank

7.6.4 Fillet surface, root radiusing

The fillet surface is that part of a tooth flank at which the usable flank merges into the bottom of the tooth space.

The root radiusing is that part of the flank profile which corresponds to the fillet surface.

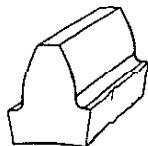


Figure 33. Fillet surface

7.6.5 Tooth tip

The tooth tip is the intersection of the flank with the crest, see Fig. 28.

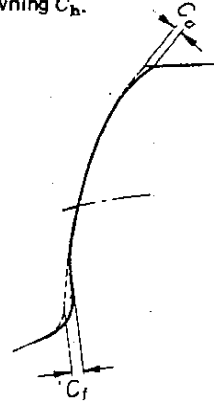
7.7 Tooth flank modifications (flank corrections)

The tooth flank modifications according to Sections 7.7.1 to 7.7.5 are flank corrections, i. e. intended deviations from the theoretical flank form directed towards the inside of the tooth. Their purpose is to compensate for disturbances to the tooth systems resulting from deviations in manufacture or to offset inaccuracies in the mounting of the gears, or to improve running properties under load or to simplify manufacture. The flank corrections according to Sections 7.7.1 to 7.7.3 are dictated by functional factors and lie within the usable flanks, whilst those according to Sections 7.7.4 and 7.7.5 are dictated by manufacturing reasons and lie outside the usable flanks.

7.7.1 Tip relief C_a , root relief C_f ; vertical crowning C_h

Tip relief C_a (or root relief C_f) is the intentional modification of the flank profile at the tip (or root) brought about by additional removal of material. As far as possible the modified flank profile produced blends continuously into the theoretical flank profile.

Tip relief and root relief (applied singly or jointly) yield the vertical crowning C_h .

Figure 34. Tip relief C_a , root relief C_f

7.7.2 Longitudinal crowning C_b (tooth trace relief)

Longitudinal crowning C_b (tooth trace relief) is the intended deviation of the tooth flank from its theoretical form in the direction of the facewidth, so that the actual tooth traces curve inwards from the theoretical tooth traces and both ends of the tooth are relieved by the amount C_b on width b_c . The modified portions of the tooth traces merge as continuously as possible into the theoretical tooth trace form.

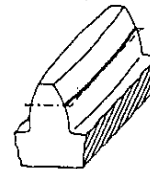


Figure 35. Longitudinal crowning

7.7.3 End relief

End relief is the reduction of the tooth thickness at one end of the tooth (one-sided longitudinal crowning).

7.7.4 Tooth tip chamfer

Tooth tip chamfer is a protective chamfer along the tooth tips which is equally divided between the crest

and the tooth tip. As a result of tooth tip chamfer the usable flank profile ends at a usable tip circle the radius of which is smaller than the tip diameter by the height of the tooth tip chamfer.

7.7.5 Grinding relief

Grinding relief is relief of the root flank (intended undercut) starting at the usable root circle and merging into the fillet surface to facilitate the finishing of the tooth flanks after the cutting of the tooth spaces. Grinding relief is generally imparted by a protuberance on the tip of the cutter tooth.

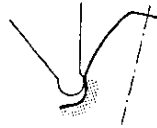


Figure 36. Grinding relief

7.7.6 Cutter interference

Cutter interference is a deviation of a flank profile from its theoretical form at the root of the tooth which can be caused by cutter interference during the cutting of the tooth system. Cutter interference usually results in shortening the usable flank profile and hence possibly in a shortening of the zone of action and weakening of the tooth root.



Figure 37. Cutter interference

7.8 Flank profiles of cylindrical gears

In the case of cylindrical gears the following flank profiles are (or were) customary. They are named according to their profile forms.

7.8.1 Involute gear teeth

In the case of involute gear teeth (see DIN 3960) the transverse profiles are portions of involutes (circular involutes) which can be thought of as being generated by the rolling of a straight line on a circle, namely the base circle, see Fig. 38. The two straight lines which contact the two base circles of a gear pair are the paths of contact for the right and left flanks; they intersect in the pitch point.

The flank profiles of the corresponding racks are straight lines which stand at right angles on the paths of contact.

Because two base circles always have two contacting straight lines and because the form of the transverse profiles is not affected by positional variations of the base circles, involute tooth systems are not sensitive to centre distance alterations in a gear pair.

7.8.2 Cycloidal gear teeth

Cycloids are curves described by the points of a "rolling circle" which rolls on or in a "pitch circle".

With double-sided cycloidal tooth systems the two rolling circles lie within the pitch circles of a gear pair, see Fig. 39. They contact each other in the pitch point and form the paths of contact for the right and left flanks.

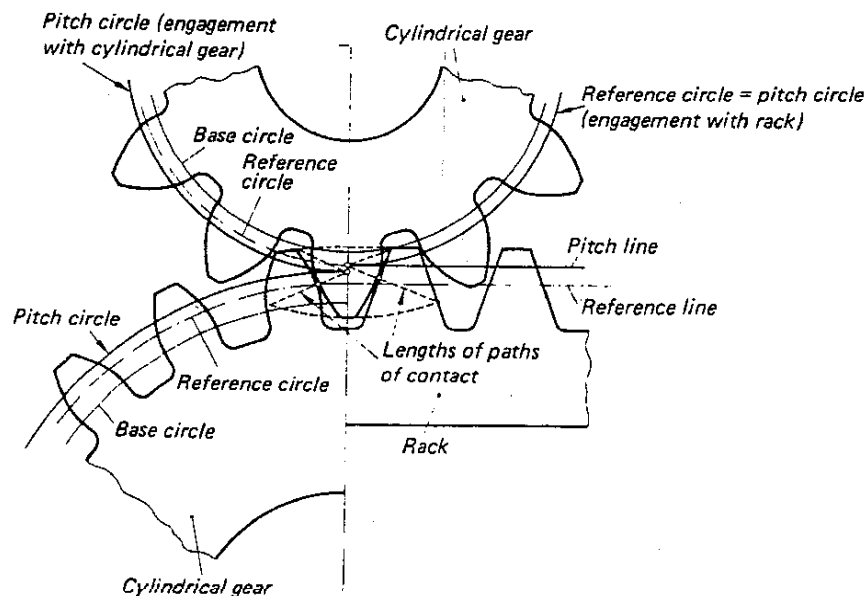


Figure 38. Involute gear teeth

In the case of external gear pairs the profile of the addendum flanks is formed by epicycloids (rolling circle rolls externally on the oppositely curved pitch circle), whilst the dedendum flanks are formed by hypocycloids (rolling circle rolls internally in a pitch circle curved in the same direction).

In the case of a rack the profiles of the addendum and dedendum flanks are orthocycloids (rolling circles roll on a straight line).

In one-sided cycloidal tooth systems there is only one rolling circle. The tooth system on one of the gears consists only of addendum flanks and on the mating gear only of dedendum flanks.

By reason of the condition that the rolling circles must pass through the pitch point cycloidal tooth systems are sensitive to centre distance variations of the gear pair.

7.8.3 Point tooth system, cylindrical lantern tooth system

The point tooth system is a single-sided cycloidal tooth system in which the rolling circle coincides with a pitch circle. The addendum flanks of one of the gears are epicycloids and the dedendum flanks of the mating gear contract to points.

For the practical implementation of these tooth systems the points are expanded to circles which take the form of driving pins (cylindrical studs; pegs, journal rollers or needles). The driving pins are arranged on the pitch circle = reference circle of the lantern gear (or on the pitch line = reference line of the lantern rack), see Fig.40. The profiles of the mating flanks arise as equidistants to the epicycloids. These tooth systems are known as lantern gear tooth systems.

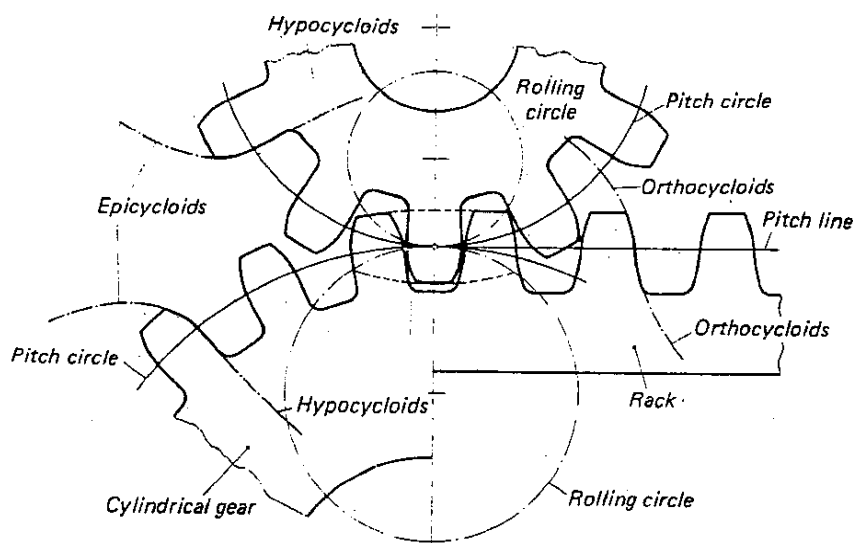


Figure 39. Double-sided cycloidal tooth system

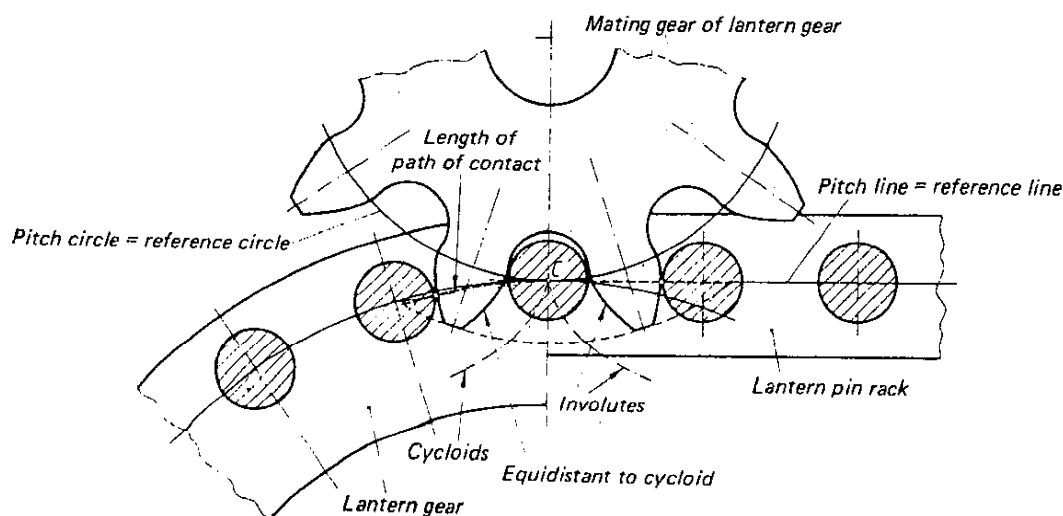


Figure 40. Cylindrical lantern tooth system

If the lantern gear carrying the pins is transformed into a lantern pin rack then the epicycloids of the mating gear and their equidistants become involutes.

7.8.4 Circular arc gear teeth

In special cases (e. g. in clockmaking for approximately true-torque transmission) circular arc teeth are also used. The flank profiles of these consist of one or two circular arcs which merge into one another.

8 Backlash and clearance between mating tooth systems

8.1 Bottom clearance c

The bottom clearance c is the minimum clearance to be specified on a normal profile between the tip surface and the root surface of gear and mating gear.

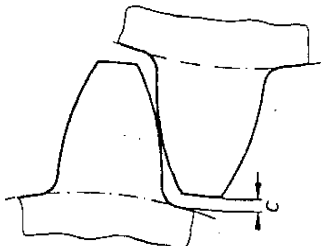


Figure 41. Bottom clearance c

8.2 Backlash j

The backlash j is the clearance between the non-working flanks of a gear pair when the working flanks are in contact with one another.

Backlash allows for pitch, profile and concentricity deviations in the two tooth systems, as well as for lubrication, temperature rise and possibly moisture (in the case of plastics gears) and it extends over the whole of the tooth flanks.

8.2.1 Circumferential backlash j_t

The circumferential backlash j_t is the length of arc to be stated on the datum surface through which each of the two gears can be rotated, whilst the other is held stationary, from the point where the right flanks are in contact to the point where the left flanks are in contact.

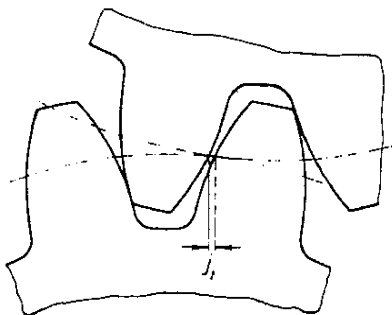


Figure 42. Circumferential backlash j_t

8.2.2 Normal backlash j_n

The normal backlash j_n is the shortest distance between the non-working flanks of a gear pair when their working flanks are in contact.

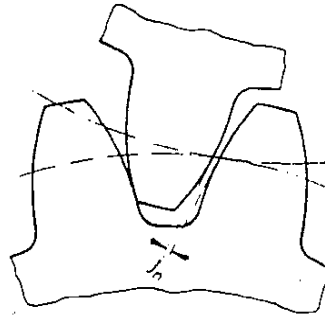


Figure 43. Normal backlash j_n

8.2.3 Radial backlash j_r

The radial backlash j_r is the difference in the centre distance or the difference in the shaft angle (in the case of a bevel gear pair) between the working condition and that of zero-backlash engagement.

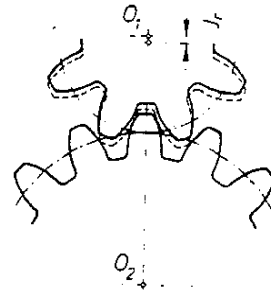


Figure 44. Radial backlash j_r

8.2.4 Axial backlash j_x

The axial backlash j_x is the amount by which a helical toothed gear can be moved along its axis, with the mating gear held stationary, from the point where the right flanks are in contact to the point where the left flanks are in contact.

8.3 Entry clearance j_e

The entry clearance j_e is the clearance existing in the transverse section with the gearing off-load which is imparted by tip relief or root relief and is effective between the driving and driven flank as they enter into engagement.

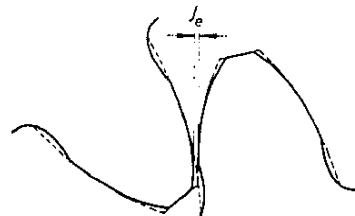


Figure 45. Entry clearance j_e

Further Standards

- DIN 3960 Definitions and specification factors for cylindrical gears and cylindrical gear pairs with involute teeth
- DIN 3960 Supplementary Sheet 1 Definitions and specification factors for cylindrical gears and cylindrical gear pairs with involute teeth; classification of the equations
- DIN 3971 Gear tooth systems; specification factors and errors relating to bevel gears; basic terms and definitions
- DIN 3975 Definitions and specification factors for cylindrical worm gear transmissions with 90° shaft angle
- DIN 3998 Part 1 Denominations on gears and gear pairs; general definitions
- DIN 3998 Part 2 Denominations on gears and gear pairs; cylindrical gears and gear pairs
- DIN 3998 Part 3 Denominations on gears and gear pairs; bevel and hypoid gears and gear pairs
- DIN 3998 Part 4 Denominations on gears and gear pairs; worm gear pairs
- DIN 3998 Supplementary Sheet 1 Denominations on gears and gear pairs; alphabetical index of equivalent terms
- DIN 3999 Symbols for tooth systems

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